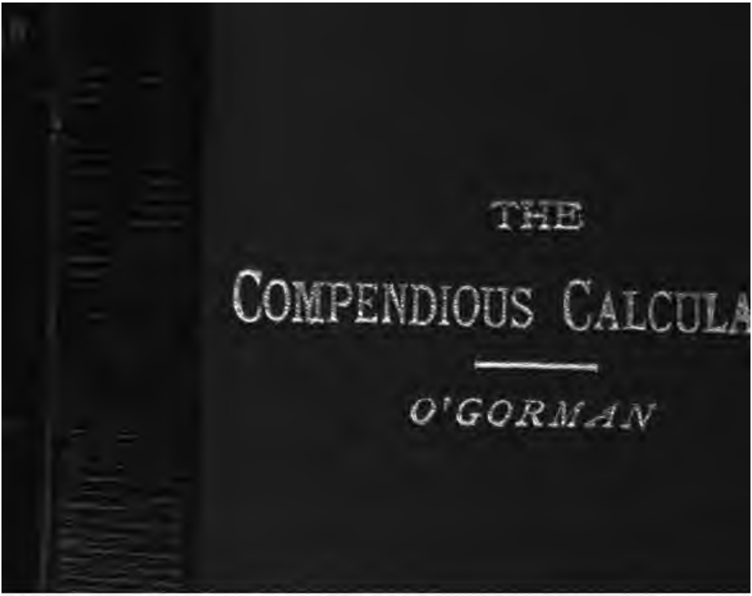
THE
COMPENDIOUS CALCULA
O'GORMAN

"*INTUITIVE CALCULATIONS*"

THE

COMPENDIOUS CALCULATOR

OR

EASY AND CONCISE METHODS OF PERFORMING THE VARIOUS ARITHMETICAL OPERATIONS REQUIRED IN

COMMERCIAL AND BUSINESS TRANSACTIONS

TOGETHER WITH

USEFUL TABLES

BY

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TWENTY-SIXTH EDITION, CAREFULLY REVISED BY

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PREFACE

TO THE TWENTY-SIXTH EDITION.

UNDER its old title, "Intuitive Calculations," this work has had a very extensive sale, having gone through twenty-five editions. But, in preparing the present edition, the Editor has thought that a more appropriate and practical title might advantageously be adopted. Hence the change of name.

It seems to have been the author's design to produce a work that should form a practically useful supplement to the ordinary treatises on common arithmetic; and, disregarding the precepts laid down in those treatises for executing, by a general Rule, every example coming under a distinct subdivision of the subject, to devise special Rules for special cases, and thus to economize figure-work as much as possible.

As a consequence of these more minute subdivisions of the subject, the Rules in this book are more numerous than those given in the school treatises. It is obviously not designed to be a school manual of arithmetic; but rather a depository of easy and expeditious methods of calculation for the guidance and use of those whose business occupations require them to be more especially expert in some particular department of the general subject: it is, in fact, in so far as its scope extends, to be regarded more as a sort of Arithmetical Dictionary, or Book of Reference, for the use of such commercial men, traders, artificers, &c.,

as may have to do with those arithmetical calculations only which are exclusively connected with their own respective callings. As these callings are special so are the Rules.

But although not intended for ordinary school purposes, yet it is a book which every teacher of arithmetic should possess. It will show him how much the exercise of common sense, and a little ingenuity, may sometimes do in the way of shortening and simplifying numerical computations; and a judicious teacher may, from time to time, avail himself of much that this work will supply, to the advantage of his pupils, in the form of oral instruction. In the opinion of the present Editor, it may be affirmed with confidence that there does not exist any work on arithmetic in which so many ingenious expedients are devised for abridging labour and saving time, and so much judicious advantage is taken of the resources of common arithmetic, as in the present volume; in fact, the book may be considered to be unique.

As a guide and book of reference it should, indeed, be indispensable to the following tradesmen, merchants, and others, viz. Wine and Spirit Merchants, Licensed Victuallers, Grocers and Tea Dealers, Bakers, Corn Merchants, Cornchandlers, and Flour Dealers, Chemists and Druggists, Wool Merchants, Jewellers, Gold and Silversmiths, Bankers and Money-changers, Stock and Share Brokers, Commission Merchants and Agents, Shippers, Carriers and Forwarding Agents, Iron Merchants, Ironfounders, Hardware Dealers and Ironmongers, Surveyors, Valuers, and Auctioneers, Contractors, Timber Merchants, Builders and Decorators, Painters, Glaziers, Paperhangers, Carpenters and Joiners, Bricklayers and Stonemasons, Paviours, Plumbers, &c., &c., each and all of whom will find in it numerous short and rapid modes of calculation suited to their several and special needs.

C. Norris.

London, *August*, 1884.

TABLE OF CONTENTS.

tion in the estimate of the work performed, or the service rendered, and the pecuniary accommodation (or credit) granted.

It is the design of this book to teach, under distinct heads, the most expeditious methods of executing the different commercial calculations here alluded to; but such of these calculations as do not admit of being reduced to a simpler or more convenient form, by any modification of the rules in common use, will be retained in their proper places here, without alteration. As to the simple and fundamental rules for Addition, Subtraction, Multiplication, and Division, we shall omit them altogether, except certain cases of the two latter, in which a departure from the common practice will be seen to be attended with advantage.

We shall merely add, in concluding these introductory remarks, that although this book is not prepared for school use, yet the judicious schoolmaster may perhaps consult its pages with profit; and in the exercise of his arduous profession may occasionally make selections from them for the special benefit of such of his pupils as he may know to be destined for specified commercial callings.

SIMPLE MULTIPLICATION.

Case 1.

When the multiplier is a number between 10 *and* 20.

Rule.—Multiply each figure of the multiplicand, in succession, by the units-figure of the multiplier, as in the common method; but here, to each product after the first partial product, add, not only what is carried, but also the next right-hand figure of the multiplicand as well: and finally, after the last partial product, add what is carried only to the last figure of the multiplicand.

Thus, in the first of the examples following, 3 times 5 are 15, and 1 is carried; 3 times 8, with this 1, are 25, which, by including the back-figure, 5, makes 30, the 3 being carried; then 3 times 3, with this carried figure, makes 12, and adding in the back-figure, 8, we get 20;

the 2, now carried to the last figure, 3, of the multiplicand, gives 5; and the work is finished.

EXAMPLES.

Multiply	385	679	4873	6958	7956	7685
by	13	16	18	17	15	19
Product	5005	10864	87714	118286	119340	146015

No explanation of this method can be needed: it is obvious that the several back-figures, added in at once with the carryings, are the figures actually written down, in the common method, and added vertically with those carryings.

CASE 2.

When the multiplier is a number between 20 *and* 30.

If the first figure of the multiplier, instead of being unit, is two units, then each back-figure is to be doubled, the rule being this:—

RULE.—Multiply as in last case, taking in the double of the back-figure; and add what is carried, from the last multiplication, to the last figure of the multiplicand thus doubled.

EXAMPLES.

Multiply	798	567	395	395	487	6784	123
by	22	23	23	27	27	28	29
Product	17556	13041	9085	10665	13149	189952	3567

To explain the foregoing, it will suffice to give the details of working the first example. Thus: twice 8 are 16, and carry 1; twice 9 are 18, and 1 (carried) are 19, and 16 (twice the back-figure, 8) are 35, and carry 3; twice 7 are 14, and 3 (carried) are 17, and 18 (twice the back-figure, 9) are 35, and carry 3; 3 (carried) and 14 (twice the back-figure, 7) are 17, and carry 1, which is to be set down in the last place, as usual. It is obvious, as in the last case, that the doubled figures, added in under the partial product arising from the next figure on the left, represent the successive multiplications by the figure

2 in the place of tens of the multiplier, which, if actually performed by the ordinary process, would be set down one place to the left and added vertically.

Case 3.

When the multiplier is 111, *or* 112, *or* 113, *&c., up to* 119.

Rule.—Multiply by the first figure on the right-hand, as in the first case, adding one back-figure in, at the second partial product; but instead of one back-figure, add the sum of the two back-figures, so soon as there are two to add; that is, when the third figure of the multiplicand is reached. Add what is carried from the final product to the sum of the last two figures, and if anything is carried from the result, add it to the last figure itself; but if nothing is carried, merely bring down this last figure.

Examples.

Multiply	2183	4296	5589	6273	7182	83716
by	111	112	113	114	115	116
Product	242313	481152	631557	715122	825930	9711056

To illustrate these operations, let us take the third example. Here 3 times 9 are 27, carry 2; 3 times 8 are 24, which, with the 2 carried, gives 26, and the back-figure 9 being taken in, we have 35, carry 3. Then 3 times 5 are 15, and 3 are 18, and 8 are 26, and 9 are 35 (8 and 9 being the two back figures), carry 3. Then 3 times 5 are 15, and 3 are 18, and 5 are 23, and 8 are 31 (5 and 8 being the two back-figures), carry 3. Again, add the 3, thus carried from the final product (31), to the sum of 5 and 5, the result is 13, carry 1, this added to the last figure (5) gives 6.

In each of the first two examples, nothing is carried to the last figure, which is therefore merely brought down.

It is readily seen that the back-figures, added in this method, are the very figures actually written down, and added vertically in the common operation.

The following general rule includes the cases 1 and 3 above, and renders special directions for them unnecessary.

General Rule.

When the multiplier is a single figure preceded by any number of units.

Rule.—1. Prefix to the multiplicand as many ciphers as there are prefixed units in the multiplier.

2. Multiply by the figure to which the units are prefixed, adding in, at the successive partial multiplications, first the single back-figure, then the two back-figures, then the three back-figures, and so on, till the back-figures thus added in are just as many in number as there are prefixed ciphers: they are never to be more in number.

3. These multiplications and additions are to be continued up to the last of the prefixed ciphers inclusive; and when this is reached the work terminates.

We shall take the fourth example in each of the two cases alluded to, and give two additional examples in which three ones are prefixed, and a final example in which four are prefixed.

Multiply	06958	006273	0006958	0006273	0000435216
by	17	114	1117	1114	11115
Product	118286	715122	7772086	6988122	4837425840

If the reader will only take the trouble to work the last of these examples in the ordinary way, he will see that the saving is considerable, as well in time as in figures; and he will at the same time perceive that the back-figures, added in, in this method, are the same as the figures actually written down, and added vertically, in the old method.

Case 4.

When the multiplier consists of three significant figures (with or without following ciphers) of which the first figure, commencing from the left, is unit, and the second is one unit greater than the third.

Underneath the multiplicand set down, as multiplier, the first and third of the significant figures as one number,

perform the multiplication, according to the preceding rule, Case 1; repeat this product on the next line underneath, but removed one place to the left; make the addition of the two lines, and the sum is the product of the multiplicand by the three significant figures of the multiplier; if these are followed by ciphers, annex as many ciphers to the result. Thus, taking the multipliers 143, 187, and the multiplicand 4681572, the work is as follows:—

```
 04681572       04681572
       13             17
 --------       --------
 60860436       79586724
60860436       79586724
---------      ---------
669464796      875453964
---------      ---------
```

*** The multipliers 143 and 187 are here virtually employed in the forms

$$\left\{\begin{array}{c}13\\130\end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{c}17\\170\end{array}\right\}$$

If ciphers had been annexed to either of the multipliers, we should of course have annexed as many to the product.

It may be added, that however numerous the figures of any multiplier may be, yet that to every set of three, coming under the above conditions, the method here proposed may be applied, care being taken to set down the successive partial multiplications in their proper places, underneath those that have preceded them, relatively to the place of units. Thus: —

```
    04681572           01234560
      143187            1541320
    --------           --------
    79586724           1481472
   79586724           1481472
  60860436          1728384
 60860436          1728384
------------      -------------
670340249964      1902852019200
```

In the last example the two triads are 132, and 154, and *the* successive multipliers, 12, 120, 14,000, and 140,000; *ciphers being neglected* excepting as to the positions *assigned to the partial* products.

To Multiply by any Number of Nines.

Rule.—Add as many ciphers to the right-hand of the multiplicand as there are nines in the multiplier, and from the result subtract the original multiplicand, the remainder will be the product.

Examples.

1.—Multiply 2368 by 999.

	2368000
	2368
Product	2365632

2.—Multiply 37568 by 999999

	37568000000
	37568
Product	37567962432

Instead of the proposed multiplier, if we were to multiply by 1, followed by as many ciphers as there are nines in the true multiplier, it is obvious that the product would be too much by the multiplicand taken once. But by annexing the ciphers, as above, we do this in effect; so that once the multiplicand has to be subtracted in order that the true product may be obtained.

LONG DIVISION.

The operation for short division, that is, when the divisor is only a single figure or digit, is too simple, by the common method, to be susceptible of, or to need, any abbreviation; but when the divisor consists of two or more digits much greater compactness may be given to the work by conducting it in accordance with the following rule.

Rule.—1. Place the divisor to the left of the dividend, as in the ordinary arrangement, and draw a horizontal at some little distance below the dividend.

2. As in the common method, find how often the divisor is contained in the number given by the first few figures of the dividend,—two, three, or four, &c., as may be found necessary; and write the corresponding quotient-figure below this horizontal line, and directly under the last of the dividend figures used.

3. Multiply *the divisor* by this quotient-figure, subtract *the result from the* used figures of the dividend, and place

the figures of the remainder, one below another, as they arise, vertically under the next unused figure of the dividend.

4. The figures, thus in vertical column (above the horizontal line), when read upwards, give the next number to which the division is to be applied; and, as before, the new quotient-figure is to be written below the horizontal line, beside the first; and, also as before, the figures of the remainder, one after another, are to be written vertically under the next figure of the dividend; and so on, as in the following examples.

Examples.

1.—786547632÷14

```
14) 78|6|5|4|7|6|3|2|
      |8|2|1|2|3|0|5|
      |.|.|1|.|1|1|.|
   ---|-|-|-|-|-|-|-|----
Quotient 5|6|1|8|1|9|7|3|  10 rem.
```

2.—237869547÷17

```
17) 23|7|8|6|9|5|4|7|
      |6|6|5|3|5|4|0|
      |.|1|1|.|.|.|1|
   ---|-|-|-|-|-|-|-|---
Quot. 1|3|9|9|2|3|2|6| 5 rem.
```

In each of these examples, the two leading figures form a number which suffices for the first partial dividend. Taking the first of these examples, this number is 78, which contains 14, 5 times, leaving the remainder 8, which is placed vertically under the 6, the next figure of the dividend. The partial dividend is now 86, which contains 14, 6 times, leaving the remainder 2. We now have 25, which contains 14 once, leaving the remainders 1 from the 5 and 1 from the 2. The next partial dividend is therefore 114, which contains 14, 8 times, giving 2 for remainder; and 27 contains 14 once, giving a remainder 3 from the 7, and 1 from the 2. The dividend now is 136, containing 14, 9 times, leaving for remainders 0 from the 6 and 1 from the 3. The dividend, 103, gives 7 for quotient and 5 for remainder; and, lastly, 52 gives 3 for quotient and 10 for remainder.

Of course the vertical lines, drawn above, would not be introduced in actual practice: they are marked here only the more clearly to show what figures are to be kept in column.

3.

```
18) 395816432
      3767403
      1111
    ---------
Quot. 21989801, 14 rem.
```

4.

```
19) 437652789
      5068603
           11
    ---------
Quot. 23034357, 6 rem.
```

5. 28) 3147869754
36160557
1 12 2

Quot. 112423919, 22 rem.

6. 32) 476987546
5810792
12 21

Quot. 14905860, 26 rem.

This method is given here chiefly on account of its conciseness: whether or not the computer can save time by employing it, will depend upon his degree of expertness in performing the two processes of multiplication and subtraction at once. A different arrangement, involving the same double process, is that exhibited in the second working of the example below, in which, however, more figures have to be written down.

7. 234) 78678543654
45472169
84577215
1 2

Quot. 336233092, 126 rem.

234)78678543654 (336233092
847
1458
545
774
723
2165
594
126 rem.

The process in either case is a combination, in mental operation, of multiplication, addition, and subtraction. Thus, by inspection, it is seen that the first partial dividend comprises the three figures on the left 786, which contain the divisor three times. Then 3 times 4 are 12, from 16 (borrowing 1), leaves remainder 4; 3 times 3 are 9, and 1 (borrowed) are 10, from 18 (borrowing 1), leaves 8; 3 times 2 are 6, and 1 (borrowed) are 7, which taken from 7 leaves no remainder. The next partial dividend, 847, contains (by inspection), the divisor 3 times, giving the second figure of the quotient. Then 3 times 4 are 12, from 17, leaves 5; 3 times 3 are 9, and 1 are 10, from 14, leaves 4; 3 times 2 are 6, and 1 are 7, from 8, leaves 1; giving 1,458 as the next partial dividend; and so on. The difference between the two arrangements consists simply in the method of writing down the successive remainders which go to form the next partial dividend.

It will be seen that each partial quotient, in the second operation, formed from the remainder and new figure brought down, is written horizontally, while in the first

operation the same partial quotient is written vertically, and the bringing down of the successive figures of the dividend is thus dispensed with. A sufficient reason for the first operation is seen by comparing it with the second. Instead of writing the several quotient-figures below a horizontal line, drawn at some distance beneath the dividend, we may write them immediately under the dividend; placing the figures of the remainder, at each step, vertically over the next unused figure of the dividend; as in the example here annexed. In this arrangement, we dispense with the necessity of estimating the distance below the dividend at which the horizontal line should be drawn; and, moreover, the divisor, the dividend, and the quotient, have the same relative positions as they have in short division. The learner may work the preceding examples in this manner, and choose whichever he finds the more convenient arrangement.

```
    1   121
    4 81404
21)567854635
 ------------
  27040696, 19 rem.
```

REDUCTION.

Money, Weights, and Measures.

Reduction is the name given to those arithmetical operations by which a quantity of one denomination is converted into another quantity of different denomination, but of the same value: the operation, for instance, by which pounds, in money, are converted into their equivalent in shillings, or pence, or farthings: hundred-weights or tons, into pounds, ounces, &c.; or lengths, such as miles or furlongs, into yards, feet, and inches. This reducing of higher denominations into lower is sometimes called *Reduction descending*; while the contrary operation, for converting the lower denominations into higher, as pence into pounds, ounces into hundred-weights, feet into miles, &c., is called *Reduction ascending*.

In money calculations it is necessary that the computer should have what is called the *Pence Table* at his fingers' ends.

Pence Table.

	s.	d.		s.	d.		s.	d.
12 *pence* are	1	0	60 *pence* are	5	0	108 *pence* are	9	0
20 „	1	8	70 „	5	10	110 „	9	2
24 „	2	0	72 „	6	0	120 „	10	0
30 „	2	6	80 „	6	8	130 „	10	10
36 „	3	0	84 „	7	0	132 „	11	0
40 „	3	4	90 „	7	6	140 „	11	8
48 „	4	0	96 „	8	0	144 „	12	0
50 „	4	2	100 „	8	4	150 „	12	6

It should also be kept in remembrance that

960 *farthings* = 240 *pence* = *one pound*, or 20*s*.

Reduction of Money.

General Rule.—All higher denominations are reduced to lower by multiplication; and all lower to higher by division. The pounds, multiplied by 20, are reduced to shillings; the shillings, multiplied by 12, to pence; and these are reduced to farthings by multiplying them by 4 On the other hand, farthings are reduced to pence by dividing by 4; pence to shillings by dividing by 12; and shillings to pounds by dividing by 20. In these operations the final denomination sought is reached by passing regularly through all the intermediate denominations in succession, but in certain cases of frequent occurrence the final denomination may be arrived at by a single step, as in the following examples.

Examples.

1. In £478 how many pence?

```
      240 (No. of pence in £1)
     ------
Ans. 114720d.
     ------
```

(See page 3.)

2. In £478 how many farthings?

```
      960 (No. of farth. in £1.)
     ------
     28680
     4302
     ------
Ans. 458880f.
     ------
```

Or shorter thus:

```
478000
 19120
------
458880f.
------
```

Here we have multiplied the 478 by 1000, which exceeds 960 by 40; and have then corrected the product by subtracting 40 times the 478.

Examples—*continued.*

3. In 114720*d.* how many pounds? (See page 11.)

```
240) 114720
         72
         89
         11
       ----
Ans. £478
```

4. In 7376640*f.* how many pounds?

```
960) 7376640
         664
         508
         683
        ----
Ans. £7684
```

Note.—In general, however, it is better to work up to the higher denomination through the lower denominations, conformably to the Rule above, since some or all of these may enter the final result.

Reduction of Weights.

To reduce a weight in one denomination to its equivalent in another denomination, whether higher or lower, certain Tables are necessary, which are different for different classes of commodities weighed: they will be inserted here under their respective heads.

Troy Weight,—Apothecaries' Weight.

24 grains (gr.)	make	1 pennyweight (dwt.)	= 24 grains
20 pennyweights (dwt.)	„	1 ounce (oz.)	= 480 grains.
12 ounces (oz.)	„	1 pound (lb.)	= 5760 grains.

As used by apothecaries, in compounding liquid medicines, the troy ounce is divided into 8 drams, and the dram into 3 scruples: the weight of the dram is therefore 60 grains, and the weight of the scruple 20 grains.

The following examples sufficiently show how this table is to be used, without any formal rule.

Examples.

1. How many grains are there in 24 lb?

```
24 lb.
12
----
288 oz.
 20
----
5760 dwt.
  24
------
138240 gr.
```

Or (see case 2, page 3.)

```
  5760 gr. in 1 lb.
    24
------
138240 gr. in 24 lb.
```

2. In 14 lb. 11 oz. 19 dwt. 16 gr., *how many gr.?*

Here the weight is 15 lb. all but 8 gr., so that $5760 \times 15 - 8 = 86400 - 8 = 86392$ gr.

3. How many spoons, each weighing 4 oz. 10 dwt., can be made out of 2 lb. 4 oz. 6 dwt. of silver?

As the weight of the silver is 28 oz. 6 dwt., it is plain that not more than six spoons can be made, the weight of which is 27 oz.: hence there are 1 oz. 6 dwt. to spare.

EXAMPLES—*continued.*

4. In 92517 gr. how many lb. troy?

$$\begin{array}{r|l} 3 & 92517 \\ 8 & 30839 \\ 2,0 & 3854;\ 7\times3=21 \text{ gr.} \\ 12 & 192;\ 14 \text{ dwt.} \\ & 16 \text{ lb. } 0 \text{ oz. } 14 \text{ dwt. } 21 \text{ gr.} \end{array}$$

Here we divide by 3 and by 8, instead of by 24, for convenience. The remainder 7, from the division by 8, is not seven grains, but 7 of the units in 30839, each of which is 3 grains, or three of the units in 92517, as is obvious: hence the 7 denotes 7 × 3 grains.

If we had divided by 8 first, and then by 3, we should have got from the first operation the remainder 5; these are of course 5 grains: the remainder from the second division would have been 2, each unit being eight grains: the complete remainder is therefore twice 8 grains and 5 grains, that is, 21 grains.

5. How many pounds troy are there in 138240 grains? *Ans.* 24 lb.
6. In 14 lb. 11 oz. 1 dwt. 16 gr., how many grains? *Ans.* 85960 gr.
7. In 75 lb. 11 oz. 19 dwt. 23 gr., how many grains? *Ans.* 437759 gr.
8. In 16 lb. 0 oz. 14 dwt. 21 gr., how many grains? *Ans.* 92517 gr.
9. How many pounds troy are there in 176360 grains?
   Ans. 30 lb. 7 oz. 8 dwt. 8 gr.
10. How many pounds troy are there in 85963 grains?
    Ans. 14 lb. 11 oz. 1 dwt. 19 gr.
11. How many grains are there in eight silver teapots, each weighing 3 lb. 9 oz. 18 dwt. 13 gr.? *Ans.* 176360 gr.
12. In 7 oz. 5 dr. 3 scr., how many scruples? *Ans.* 186 scr.
13. How many pounds are there in 4896 scruples? *Ans.* 17 lb.
14. A patient takes 2 dr. 2 scr. of bark daily: how long will 7 lb. last him? *Ans.* 252 days.
15. What weight of gold will be required to make twelve ornaments, each weighing 1 oz. 18 dwt. 12 gr. *Ans.* 23 oz. 2 dwt.

NOTE.—In this last example, we see that the weight of each ornament is 2 oz. less $1\frac{1}{2}$ dwt.; so that the weight of the twelve ornaments must be 24 oz. diminished by 12 times $1\frac{1}{2}$ dwt.; that is, by 18 dwt., hence the weight is 23 oz. 2 dwt., which is thus easily determined without putting pen to paper. There are many similar cases in all arithmetical processes where the results may be determined, or the operation shortened, by mere inspection.

GOLD AND SILVER COINS.

The coins, gold and silver, in present use in the United Kingdom, are the following.

Gold Coins.

Sovereign,	in value	20*s.*,	in weight	5 dwt. 3·274 gr.
Half-sovereign,	,,	10*s.*,	,,	2 dwt. 13·637 gr.

The above decimals are respectively 274 thousandths,

and 637 thousanths of a grain. The amount of pure gold in a sovereign is (or should be) 113 grains and one-thousandth of a grain; but the whole weight of a new sovereign, expressed in grains, is 123 grains and 274 thousandths of a grain.

The coins are not entirely of pure gold, a metal which is too soft, and therefore too easily bruised and battered, to be well adapted for use, in its pure state, as money. It is sufficiently hardened by being mixed with an alloy, to the amount of one-twelfth ($\frac{1}{12}$) of the whole weight, of copper; so that the gold coin has only $\frac{11}{12}$ of its weight pure gold. A mass of this mixed metal is called Mint Gold or Standard Gold, and is said to be 22 carats fine; which means that if the mass, of whatever weight, be divided into 24 equal parts, or carats, 22 of those parts only will express the weight of pure or fine gold, and the other two parts the weight of copper, in the mass; so that into every pound troy of Standard Gold there enters one ounce of alloy. The Mint price of this standard gold is £3 17*s.* 10½*d.* per ounce, or £46 14*s.* 6*d.* per pound; a pound of it is therefore coined into 46 $\frac{29}{40}$* sovereigns, or 40 lb. into 1869 sovereigns.

⁎ The legal standard for gold watch-cases is 18 carats fine. What is called the *Hall mark* (a crown and the figures 18), stamped by the authority of the Goldsmiths' Company on the case, is a warrant for this degree of purity, namely, that one-fourth part only is alloy.

Silver Coins.

Crown-piece,	in value 5*s.*,	weight, 18 dwt.,	4·3636 gr.	or 18 dwt.	$4\frac{4}{11}$ gr.
Half-crown,	,, 2*s.* 6*d.*	,, 9 ,,	2·1818	,, 9 ,,	$2\frac{2}{11}$,,
Florin,	,, 2*s.* 0*d.*	,, 7 ,,	6·5454	,, 7 ,,	$6\frac{6}{11}$,,
Shilling,	,, 12*d.*	,, 3 ,,	15·2727	,, 3 ,,	$15\frac{3}{11}$,,
Sixpence,	,, 6*d.*	,, 1 ,,	19·6363	,, 1 ,,	$19\frac{7}{11}$,,
Four-penny-piece	,, 4*d.*	,, 1 ,,	5·0909	,, 1 ,,	$5\frac{1}{11}$,,
Three-penny-piece	,, 3*d.*	,, 0 ,,	21·8181	,, 0 ,,	$21\frac{9}{11}$,,

In standard silver for coinage there are $\frac{37}{40}$ths of the whole of pure silver, and the remaining $\frac{3}{40}$ths of alloy; so that a pound of standard silver contains 11 oz. 2 dwt. of fine silver, and 18 dwt. of alloy; that is, 2 dwt. less than

* 14*s.* 6*d.*, expressed as a fraction of £1, is $£\frac{14\frac{1}{2}}{20} = £\frac{29}{40}$.

an ounce: it is coined into 66 shillings, its Mint price being at the rate of 66*d*., or 5*s*. 6*d*. an ounce.

NOTE.—As already observed above, the word *carat*, when used in reference to the purity of the precious metals, denotes merely the twenty-fourth part of the entire mass; but the same term, when employed in reference to the weight of diamonds, stands for $3\frac{1}{5}$ grains. A diamond of the first water, that is, of the first quality, when without flaw and properly cut, is worth £8 if it weigh 1 carat; it is worth four times as much, or £32, if it weigh 2 carats; nine times as much, or £72, if it weigh 3 carats; sixteen times as much, or £128, if it weigh 4 carats; and so on, the worth being estimated at £8, multiplied by the square of the number of carats.

By Act of Parliament "all articles sold by weight shall be by avoirdupois weight, except gold, silver, platina, diamonds, and other precious stones, and drugs when sold by retail, and that such excepted articles, and none others, may be sold by troy weight."

Apothecaries, though always compounding their medicines by troy weight, yet buy and sell the ingredients by avoirdupois.

AVOIRDUPOIS WEIGHT.

16 drams	make	1 ounce = $437\frac{1}{2}$ grains.
16 ounces	,,	1 pound = 7000 ,,
14 pounds	,,	1 stone.
28 pounds	,,	1 quarter of a cwt. (hundred-weight).
4 quarters	,,	1 cwt. = 112 lb. = 8 stone.
20 cwt.	,,	1 ton = 2240 lb.

NOTE.—1 pound avoirdupois is equal to 14 oz. 11 dwt. $15\frac{1}{2}$ gr. troy; so that if a person were to get a pound of any commodity, weighed by the troy-pound-weight, he would get less than if the article were weighed by the avoirdupois-pound-weight: but if he were to receive an ounce, weighed by the troy-pound-weight, he would get more than if it were weighed by the avoirdupois-pound-weight; for an avoirdupois-ounce is only $437\frac{1}{2}$ grains, whilst a troy-ounce is 480 grains, the grain being the same weight in both cases.

Formerly the stone varied in different parts of the kingdom, from 8 lb. to 16 lb.; but by an Act of Parliament, passed in 1834, the legal stone was fixed at 14 lb.: nevertheless, butchers, in London and the suburbs, use a stone of 8 lb. for meat.

Besides the above denominations used in avoirdupois weight, there are several others peculiar to the particular class of commodities weighed: thus the following denominations are employed for wool.

Wool Weight.

	Av. lb.		Av. lb.
1 clove = $\frac{1}{2}$ a stone =	7	1 sack = 2 weys =	364
1 tod = 2 stone =	28	1 last = 12 sacks =	4368
1 wey = $6\frac{1}{2}$ tod =	182	= 39 cwt.	

And for hay and straw, the additional terms *truss* and *load* are used thus:—

Hay and Straw.

	Av. lb.		cwt. lb.	Av. lb.
1 truss of straw	= 36	1 load of old hay	= 18 0	= 2016
1 „ old hay	= 56	1 „ new hay	= 19 32	= 2160
1 „ new hay	= 60	1 „ straw	= 11 64	= 1296

It will be seen from this table that 36 trusses go to a load, whether they be of straw or hay, new or old; so that the term *load* here does not imply a fixed weight or number of pounds, but only a fixed number of trusses, namely, 36.

There are several other articles of merchandize, of which certain weights carry particular names: the following are the principal of these.

Miscellaneous Articles.

	Av. lb.		Av. lb.
A firkin of butter	= 56	A puncheon of prunes	= 1120
A „ soap	= 64	A bushel of flour	= 56
A barrel of „	= 256	A sack of flour	= 280
A „ anchovies	= 30	A fother of lead, 19 cwt. 2 qr.	= 2184

A sack of coal weighs 2 cwt., or 224 lb.; so that 10 sacks make a ton.

The following are compendious methods of reducing hundred-weights, quarters, and pounds, to pounds.

RULE I.—Multiply the cwts. by 12, adding in the overplus weight reduced to pounds. Write the result under the cwts., so that the place of hundreds may be directly under the place of units of the cwts., and then add. Or

RULE II.—Repeat the number of cwts. under itself; repeat again, this time removing the figures one place to the left; *repeat still again,* removing the figures one place more to *the left; then add* the four rows up, taking in the odd *pounds that are over* and above the cwts.

The following examples will practically illustrate both these short and convenient rules.

Examples.

1. How many pounds are there in 123 cwt. 3 qr. 10 lb.?

By the first rule, we have to multiply the 123 by 12, taking in the 3 qr. 10 lb. in pounds, namely, 94 lb., placing the result, 1570, as directed, thus—

	cwt.	lb.		cwt.	lb.		cwt.	lb.
By Rule I.	123	94	By Rule II.	123	94	or,	123	94
	1570			123			123	
				123			1234	
Ans.	13870 lb.			12394			1239	
			Ans.	13870 lb.		*Ans.*	13870 lb.	

This last operation differs from that immediately preceding it only as to the manner of disposing of the odd 94 lb. It is mere matter of taste which arrangement be adopted.

2. How many pounds are there in 26 cwt. 1 qr. 13 lb.?

	cwt.	lb.		cwt.	lb.		cwt.	lb.
By Rule I.	26	41	By Rule II.	26	41	or,	26	41
	353			26			26	
				26			261	
Ans.	2953 lb.			2641			264	
			Ans.	2953 lb.		*Ans.*	2953 lb.	

3. In 14 tons 17 cwt. 0 qr. 3 lb., how many pounds?

As 20 times 14 are 280, the weight in cwts. and lbs. is therefore 297 cwt. 3 lb.

	cwt.	lb.		cwt.	lb.
By Rule I.	297	3	By Rule II.	297	3
	3567			297	
				2973	
Ans.	33267 lb.			297	
			Ans.	33267 lb.	

The explanation of these rules is as follows: take the last example. Multiplying 297 by 112 is evidently the same as multiplying it first by 100, and then adding 12 times 297 to the result. But this result is 29700, and 12 times 297 is 3564; and these are the two numbers which, with the 3 lb., are added together above, conformably to *the first rule.*

Again. In multiplying 297 by 112, in the usual way, we take 297 twice, thus getting 594, under which 297 is afterwards twice written down, but the figures are removed each time a place to the left, exactly as above.

The first of these rules involves multiplication by 12, the second requires addition only. We shall add two examples of the reverse operation, the bringing of lbs. into cwts.

4. In 2953 lb. how many cwt.? (See page 7 for rule for Division.)

```
112) 2953
        1
        7
     ----
     26 cwt. 41 lb.
   = 26 cwt. 1 qr. 13 lb.
```

5. In 13870 lb. how many cwt.?

```
112) 13870
        63
        24
      ----
      123 cwt. 94 lb.
    = 123 cwt. 3 qr. 10 lb.
```

6. How many lbs. are there in 75 cwt. 3 qr. 14 lb.? *Ans.* 8498 lb.
7. How many lbs. are there in 976 cwt. 3 qr. 27 lb.? *Ans.* 109423 lb.

This example will of course be computed for 977 cwt., and then 1 lb. deducted from the result. In like manner the weight in the preceding example may be regarded as 76 cwt., and 14 lb. be deducted afterwards.

8. In 3 cwt. 2 qr. 14 lb. of sugar, how many half-pound parcels are there? *Ans.* 812.
9. In 264 cwt. 3 qr. 12 lb. 11 oz. how many oz.? *Ans.* 474635 oz.
10. In 249901 oz. how many cwt.? *Ans.* 139 cwt. 1 qr. 22 lb. 13 oz.
11. How many pounds are there in 24 bags of flour, each weighing 2 cwt. 2 qr. 13 lb.? *Ans.* 7032 lb.

Some articles are sold wholesale (by *tale*) by what is called the *Long Hundred*; that is, by the six score or 120, instead of by the five score or 100. The following is a general rule for converting hundreds of the one kind into hundreds of the other kind.

To reduce common hundreds to long hundreds, and the contrary.

RULE.—From the number of common hundreds subtract the sixth part of that number: the remainder will be the number of long hundreds.

To the number of long hundreds add the fifth part of

that number: the sum will be the number of common hundreds: thus—

1. 6) 468 common hundreds. 78 — 390 long hundreds.	2. 5) 390 long hundreds. 78 — 468 common hundreds.

The reason is obvious: there will be only $\frac{5}{6}$ as many long as there are common hundreds, seeing that $\frac{5}{6}$ of 1 long hundred = 1 common hundred; and $\frac{5}{6}$ of any number is that number *minus* $\frac{1}{6}$ of it. Also there must be $1\frac{1}{5}$ as many common hundreds as there are long hundreds, since $1\frac{1}{5}$ of the former make but 1 of the latter; and $1\frac{1}{5}$ of any number is that number *plus* $\frac{1}{5}$ of it. The examples following will suffice for exercise in this rule.

3. How many long hundreds are there in 320 common hundreds? *Ans.* $266\frac{2}{3}$.
4. How many common hundreds are there in 256 long hundreds? *Ans.* $307\frac{1}{5}$.
5. How many long hundreds are there in 24000? *Ans.* 200.
6. How many common hundreds are there in 173 long hundreds? *Ans.* $138\frac{2}{5}$.

The answer to example 3 implies that there will be 266 long hundreds, and 80 individual articles besides; the answer to the 4th example shows that there will be 307 common hundreds, and 20 articles over; and the answer to example 6 shows that there will be 138 common hundreds, and 40 articles besides.

Reduction of Measures.

Measures, like weights, are reduced to equivalent measures, having other denominations, by the aid of one or other of the following tables.

I. Measures of Length, or Long Measure.

The principal measures of length used in this kingdom are these:—

12 inches	make	1 foot.
3 feet	,,	1 yard.
$5\frac{1}{2}$ yards	,,	1 rod, pole, or perch.
40 poles	,,	1 furlong = 220 yards.
8 furlongs	,,	1 mile = 1760 yards = 5280 feet.

The old Scotch and Irish miles are respectively $1\frac{1}{8}$ and $1\frac{3}{11}$ English, so that 8 Scotch miles are equal to 9 English, and 11 Irish to 14 English.

Surveyors measure land by the chain, consisting of 100 links. The length of the chain is 4 poles, or 22 yards* = 66 feet, so that the length of a single link is the 100th part of 66 feet, that is, $7\frac{92}{100} = 7{\cdot}92$ inches. Distances both at land and sea are sometimes measured in leagues, the league being a length of 3 miles. It should be mentioned, however, that the land mile and league are not the same as the nautical mile and league; the nautical mile (the 60th part of a degree of the equator) exceeds the land mile: it is 6086 feet; and the nautical league is three such miles. Sea-depths or soundings are measured in fathoms, the fathom being 6 feet.

For the height of horses, the unit of measure is the hand = 4 inches; so that a horse fifteen hands high is 5 feet in height.

Drapers and mercers use the measures, ell, yard, inch, and nail, but not the foot. The measures are as follow:—

Cloth Measure.

$2\frac{1}{4}$ inches	make	1 nail.
4 nails	„	1 quarter (of yd.)
4 quarters	„	1 yard.
5 quarters	„	1 English ell, and
6 quarters	„	1 French ell.

NOTE. In early times the inch was estimated as the length furnished by putting together three grains of barley, end to end; and that "three barley-corns make one inch," is a statement still retained in many tables of linear measure.

To reduce miles to yards, and the contrary.

RULE I.—Multiply 1760 by the number of miles; but if this number consist of three or more figures, it will usually be found more convenient to work by one or other of the two rules following.

* This length is chosen for the surveyor's chain, for convenience in the calculation of acres of surface: an acre being 4840 square yards, it *is equal to 10 times* a square chain, that is, to 10 square chains.

RULE II.—Multiply the number of miles by 44, then the result by 4, and annex a 0 to the final product. Or:

RULE III.—Multiply the number of miles by 16 in one line (Case 1), repeat this line of figures underneath, commencing one place back to the left, and then add up the two rows of figures, annexing 0 to the result (*v.* Case 4, page 5).

By either of these rules the miles will be reduced to yards. To convert yards into miles we must divide by 1760.

EXAMPLES.

1. How many yards are there in 374 miles?

By Rule I.	By Rule II.	By Rule III.
374	374	374
1760	44	16
22440	1496	5984
6358	1496	5984
Ans. 658240 yds.	16456	*Ans.* 658240 yds
	4*	
	Ans. 658240 yds.	

2. How many yards are there in 2683 miles?

By Rule I.	By Rule II.	By Rule III.
2683	2683	2683
1760	44	16
160980	10732	42928
45611	10732	42928
Ans. 4722080 yds.	118052	*Ans.* 4722080 yds.
	4*	
	Ans. 4722080 yds.	

The operation by Rule I. requires no explanation: the multiplication by the 17 is performed in one line. According to Rule II., the multiplication is by 44, 4, and 10; and $44 \times 4 \times 10 = 1760$. By Rule III., the number 176 is split into the two numbers 16 and 160: the first row of figures in the work arises from multiplying by the 16, and

* This multiplier may of course be used first, instead of last; that is, we may multiply four times the number of miles by 44.

the second row being 10 times the first, arises from the multiplication by the 160: the sum of the two rows is therefore the product arising from the multiplier 176, and 0 annexed to this gives the product resulting from the multiplier 1760.

Although in multiplying by so small a number as 1760, the trouble and risk of error by the ordinary method are but little, yet by either of the methods II., III., both are still less. It may be worthy of notice too that the three 4s, employed in the second rule, may serve to recall the number of yards in a mile, should it escape the memory.

3. How many miles are there in 658247 yards, and in 4722084 yards, respectively? [See Rule for Division, p. 7.]

```
176,0) 65824,7          176,0) 472208,4
         00 0                    062 0
         37 0                    245 0
         1                       11
        ----                     ----
  Ans. 374 miles 7 yds.   Ans. 2683 miles 4 yds.
```

4. How many yards are there in 3057 miles? *Ans.* 5380320 yds.
5. How many miles are there in 1793440 yards? *Ans.* 1019 miles.
6. How many feet are there in the earth's diameter at the equator, its measure in miles being 7925·648 miles? *Ans.* 41847421·44 feet.

In this example we are required to reduce miles to feet. By the usual method of operating, we should multiply the number of miles by 5280, the number of feet in a mile. But the trouble will be somewhat lessened by employing the principle of Rule III. The number $528=44\times3\times4$; and $44\times3=132$; so that, by the principle referred to, we may use the multipliers $\left\{\begin{matrix}12\\12\end{matrix}\right\}$ and 4: we shall here exhibit the operation in juxtaposition with that by the common rule.

```
     7925·648                 7925·648
         5280                       12
    ---------                 --------
    634051840                 95107776
   15851296                  95107776
  39628240                   ----------
  ---------                  1046185536
Ans. 41847421·440 feet.               4
  ---------                 ----------
                     Ans. 41847421·440 feet.
```

As the terminating figures here are decimals, the final cipher is of course superfluous. It is inserted merely that in the second method of working the principle explained above may be strictly conformed to.

There is the same amount of figure-work in the second as there is in the first of these operations; but there is unquestionably some saving of head-work in the second mode of proceeding, and it is this kind of saving which is the main object of consideration in forming an estimate of a short method.

II. Superficial or Square Measure.

Table I., at page 19, is applied exclusively to the measure of lengths: this table is applied exclusively to the measure of surfaces.

144 square inches make 1 square foot.
9 square feet make 1 square yard.
100 square feet make 1 square.
$272\frac{1}{4}$ square feet ($30\frac{1}{4}$ sq. yds.) make 1 square rod, pole, or perch.
40 square rods, or poles, make 1 rood=1210 sq. yds.=$2\frac{1}{2}$ sq. chains.
4 roods (4840 sq. yds.) make 1 acre=10 square chains.
640 acres make 1 square mile=6400 sq. chains.

Hence a piece of land which is equivalent in surface to a square each side of which is 10 chains, measures 10 acres; and a piece equivalent in surface to a square of which each side is 80 chains, measures a square mile, since $80\times80=6400$.

III. Cubic or Solid Measure.

This measure is used whenever three dimensions, length, breadth, and thickness (or depth) are all taken into consideration. A cube is a figure in the form of the gambler's die, the three dimensions being all equal. When each dimension is an inch, the matter or space is a cubic inch of that matter or space; when each dimension is a foot, it is a cubic foot, and so on.

1728 cubic inches	make	1 cubic foot.
27 cubic feet	„	1 cubic yard.

A cubic yard of earth is reckoned to be 1 load.

*** These tables will come into application hereafter, in the calculations of artificers' work.

MEASURES OF CAPACITY.

I. Dry Measure.

(Chiefly for corn, meal, flour, peas, beans, &c.)

2 pints	make	1 quart.	4 bush.	make	1 coomb.
4 quarts	,,	1 gallon.	2 coombs	,,	1 quar. = 8 bush.
2 gallons	,,	1 peck.	5 quarters	,,	1 wey or load.
4 pecks	,,	1 bushel.	2 weys	,,	1 last, or 10 qrs.
2 bushels	,,	1 strike.	A pottle is half a gallon.		

*** The gallon measure contains 277·274 cubic inches.

II. Wine and Spirit Measure.

4 gills (or qtns.)	make	1 pint.	54 gallons	make	1 hogshead.
2 pints	,,	1 quart.	2 hogsheads	,,	1 pipe or butt.
4 quarts	,,	1 gallon.	4 hhds. (2 pipes)		1 tun.

Note.—The last three terms in this table—the terms hogshead, pipe, and tun, are names which are used more to designate the kind of casks than to denote definite measures of capacity. The measure of a pipe, namely, 108 gallons, given above, applies exclusively to sherry wine; for port, claret, madeira, and other wines, the number of gallons to the pipe is different. But it is the practice to gauge the casks which bear the before-mentioned names, and thus to ascertain the number of gallons contained in them by direct measurement.

III. Ale and Beer Measure.

2 pints	make	1 quart.
4 quarts	,,	1 gallon.
9 gallons	,,	1 firkin.
2 firkins (18 gal.)	,,	1 kilderkin.
2 kilderkins	,,	1 barrel = 36 gallons.
3 kilderkins	,,	1 hogshead = 54 gallons.
2 hogsheads	,,	1 butt = 108 gallons.
2 butts	,,	1 tun = 216 gallons.

Note.—Till the year 1826, the gallon was a measure of varying capacity, its cubic contents being different for different dry or liquid commodities measured by it: the wine gallon, the ale and beer gallon, and the corn gallon, were all of different cubic contents. The imperial gallon is now the only legal gallon, and it alone is to be universally employed throughout the British dominions as the measure bearing that name, without any distinction as to the articles measured. It is enacted that the gallon shall contain 277·274 cubic inches, and as this number of inches is the cubic measure of 10 lbs. avoirdupois of distilled water (when the temperature of the air is at 62° Fahrenheit, and the barometer stands at 30 in.), we learn that

"A pint of pure water
Weighs a pound and a quarter."

The weight, in grains, of a gallon of pure water, at the above temperature and pressure, is 70000 grains.

IV. Divisions of Time.

60 seconds	make	1 minute.
60 minutes	„	1 hour.
24 hours	„	1 day.
28, 29, 30, or 31 days	„	1 calendar month.
12 calendar months	„	1 year.
365 days	„	1 common year.
$365\frac{1}{4}$ days	„	1 Julian year.
365 days 5 hours 48 minutes 48 sec.	=	1 solar year.

The solar year is the time in which the earth makes one revolution in its orbit round the sun, or the time which elapses between the departure of the sun from the vernal equinox (or where its path crosses the equinoctial) till its return to the vernal equinox again. As the equinoctial points shift a little, it is the interval of time, on the average—the Mean Solar Year, which 365 days 5 hrs. 48 min. 48 sec. measures. This period of time is only 11 min. 12 sec., or $11\frac{1}{5}$ min., short of $365\frac{1}{4}$ days; and since, from neglecting the fraction of a day beyond the 365 days, the Roman calendar retrograded more and more from the true period of the year which it nominally indicated, Julius Cæsar caused it to be readjusted, and enacted that every fourth year afterwards should be a year of 366 days; hence our bissextile, or leap-year, in which an additional day is added to the month of February every fourth year, the other years counting as 365 days only. But as the Julian year is 11 min. 12 sec. in excess, the error would be one day in excess in 129 years, another adjustment was, therefore, afterwards seen to be necessary; and this important "*Reformation of the Calendar*," as it is called, was accomplished by direction of the enlightened Pontiff Gregory XIII., in the year 1582. As the error, in excess, of the Julian reckoning amounted to about three days in 400 years, it was enacted that the additional day added to February, in ordinary leap-years, should be omitted in the years which completed centuries, unless when these

centenary years were multiples of the number 400. The error by this adjustment becomes so small that it amounts to only a day in about 3600 years. To compensate for the accumulated errors of the Julian reckoning, the Reformed Calendar commenced with the suppression of ten of the days of the Julian Calendar, it being ordered that the 5th of October in that year (1582) should be called the 15th.

The annexed table, showing the number of days from one date to another, will occasionally be found useful; it will be sufficiently understood from an example or two of its application.

Examples.

1. Required the number of days from the 9th of May to the 17th of September in the same year. Opposite May, on the right, will be found 120, the number of days from the 1st day of January, inclusive, to the last day of April, inclusive; hence, adding the 9 days in May, we have 129 days, including the 9th of May itself. Again, opposite September, on the right, is found 243, the number of days from the 1st of January, inclusive, to the last day of August, inclusive. Adding therefore the 17 days in September, we have 260 days from January 1 to September 17, both inclusive; then subtracting the 129 from the 260, there remains 131, the number of days which follow the first of the given dates up to the second, including that second date.

DAYS.		MONTHS.	DAYS.	
31	334	January .	00	31
59	306	February.	31	28
90	275	March . .	59	31
120	245	April . . .	90	30
151	214	May	120	31
181	184	June . . .	151	30
212	153	July . . .	181	31
243	122	August . .	212	31
273	92	September	243	30
304	61	October .	273	31
334	31	November	304	30
365	00	December	334	31

Of course, if the end of a leap-year February be included in the interval, another day must be added.

2. How many days are there from the 5th of November, 1868, to the 16th of May (inclusive), 1869?

Here some of the days are in one year, and the remaining days in the next year; and we proceed thus:—

Subducting the 5 days of November, already elapsed, from the 30 days of November, there remain 25 days: add these to the 31 days, opposite November, on the left; to the sum (56) add the 120 opposite May, thus bringing the days to the end of April; then taking account of the 16 days following the end of April, we have 192 for the total number of days from the first given date to the second, including that second date.

As before, if the end of a leap-year February occur in the interval, another day must be added.

It will be observed that in the column immediately

following the column of MONTHS, the number against any month expresses the number of days of the year which have elapsed before the 1st of that month. And in the column immediately preceding the column of MONTHS, the number against any month expresses the number of days requisite, after the last day of that month, to complete the year. The first column in the table merely gives the number of days in the first month of the year, in the first two months, the first three months, and so on, up to the whole twelve months; while the last column expresses the number of days in each individual month.

[For the purpose of calling these days to mind nothing can be more suitable than the well-known doggerel:—

Thirty days hath September,
April, June, and dull November,
February hath twenty-eight alone,
And all the others thirty-one,
Except leap-year, and that's the sign
That February has twenty-nine.

or

Leap-year coming once in four,
February has one day moré.]

3. How many days are there from July 18 to December 27, inclusive, in the same year? *Ans.* 162 days.
4. How many days are there from November 17, 1868, to September 12, 1869? *Ans.* 299 days.
5. How many days are there from November 17, 1867, to September 12, 1868? *Ans.* 300 days.

DIVISIONS OF THE CIRCLE.

For certain purposes of calculation, where arcs of circles are concerned, it has been found convenient to regard the entire circumference of a circle (of whatever magnitude) as to be divided into 360 equal parts, and to call each of these parts a degree; the degree is further subdivided into 60 equal parts called minutes, and each of these again into 60 equal parts called seconds; a portion of arc still smaller than a second is expressed as a fraction or decimal of a second. An arc of, say, 23 degrees 47 minutes 13 seconds, is denoted thus, $23^\circ\ 47'\ 13''$, and

similarly in other cases. An arc of 90° is called a quadrant. It may be well to mention that, from this mode of measurement, no estimate can be formed of the actual length of arc, unless that of the entire circumference to which it belongs be stated, since the term degree does not imply any definite length, any more than the word circumference does. As the earth performs a complete rotation in 24 hours, the equator, and every circle parallel to it (every parallel of latitude), turns through a 24th part of the entire circumference, or 360°, every hour; that is, it rotates at the rate of 15° an hour. We can thus readily find the time corresponding to any given arc of the earth's rotation, or the difference of time, at the same instant, between two places whose difference of longitude is given. The rule for doing this is as follows:—

To find the time corresponding to an assigned number of degrees, minutes, and seconds of longitude.

RULE.—1. Divide the number of degrees by 15; the quotient is the number of hours.

2. Multiply the remainder (if there be a remainder) by 4; the product is the number of minutes of time.

3. Divide the minutes of arc by 15; the quotient is the number of minutes of time.

4. Multiply the remainder, if any, by 4; the product is seconds of time.

5. Divide the seconds of arc by 15; the quotient is seconds of time. The sum of these results will be the hours, minutes, and seconds corresponding to the given arc.

EXAMPLES.

1. Required the time corresponding to 108° 24′ 22″.

		h.	m.	s.
Time for	108°	7	12	0
,,	24′		1	36
,,	22″			1·47
Time for 108° 24′ 22″	. . . .	7	13	37·47

The decimal ·47 is written for the interminable decimal ·466 . . . .

That the foregoing operations must lead to the correct result is pretty obvious. The complete quotient of 108 divided by 15 is $7\frac{3}{15}$; so that $7\frac{3}{15}$ hours correspond to 108 degrees; but the fraction of an hour is converted into minutes by multiplying it by 60; and 60 times 3 divided by 15 is 4 times 3. In like manner for the 24′; $24 \div 15 = 1\frac{9}{15}$; and $\frac{9}{15} \times 60 = 4$ times 9. As to the seconds of arc, $22 \div 15 = 1\frac{7}{15}$; and by proceeding in the same way with this fraction, we should get from it 4 times 7 thirds; but as thirds of time are not used, the fraction is expressed in decimals of seconds, namely, $\frac{7}{15}$ seconds = ·4666 . . . seconds.

2. Required the time corresponding to 84° 42′ 30″. *Ans.* 5 h. 38 m. 50 s.
3. Required the time corresponding to 93° 47′ 41″.
   Ans. 6 h. 14 m. 30·72 s.
4. Required the time corresponding to 230° 32′ 10″.
   Ans. 15 h. 22 m. 8·7 s.

To find the angular measure corresponding to an assigned time.

This problem is the converse of the preceding one: the rule is this:—

RULE.—1. Multiply the number of hours by 15; the product is so many degrees.

2. Divide the minutes and seconds of time by 4, and reckon every unit of remainder as 15′, if minutes be the dividend, and as 15″, if seconds be the dividend.

EXAMPLES.

1. What angular measure (or arc) corresponds to 3 h. 14 m. 23 s.?

For 3 h. . . .	45°		
,, 14 m. . .	3	30′	
,, 23 s. . . .		5	45″
Angular Measure	48°	35′	45″

2. Find the angular measure for 2 h. 18 m. 58·26 s.

For 2 h. . . .	30°	
,, 18 m. . . .	4	30′
,, 58·26 s. . . .	14	33·9″
Angular Measure	34° 44′	33·9″

The remainder from the 58·26 seconds is 2·26; and 15 times this is 33·9.

3. Find the arc corresponding to 5 h. 19 m. 37 s. *Ans.* 79° 54′ 15″.
4. What angular measure corresponds to 2 h. 18 m. 58·27 s.? *Ans.* 34° 44′ 34″.
5. Required the degrees, minutes, and seconds, corresponding to 7 h. 13 m. 37·4666 . . . s. *Ans.* 108° 24′ 22″.

COMPOUND MULTIPLICATION,

OR MULTIPLICATION OF QUANTITIES MADE UP OF DIFFERENT DENOMINATIONS.

We commence the practical application of the particulars tabulated in the last few pages with Compound Multiplication, because the easy operations of addition and subtraction, as taught in every schoolboy course of arithmetic, admit of no simplifications or abridgments; and because, moreover, these operations will actually be involved in the working of most of the examples which follow. Previously, however, to entering upon these, we shall give a table of the factors of *Composite Numbers*, as far, at least, as the number 10000. A composite number is one that admits of decomposition into integral factors; that is, it is a number that may be produced by the multiplication together of whole numbers only. When the multiplier of a quantity made up of several denominations is a composite number of two or three figures, it is, in general, much more convenient to multiply by the factors of it, one after another, than to multiply by the higher number itself; and the Table will point out what these lower multipliers, in each case coming within its limits, are. And it will hereafter be seen that, even in the case of a non-composite, or prime number, as it is called, a reference to the table will always suggest a saving of trouble, and diminish the risk of error in the work. Those composite numbers, the factors of which are given by the multiplication table itself, are here omitted.

A TABLE of those Factors of the Composite Numbers from 75 to 10000 which fall within the Limits of the Multiplication Table.

No.	Factors.	No.	Factors.	No.	Factors.
75	5 5 3	405	9 9 5	1029	7 7 7 3
98	7 7 2	432	12 9 4	1056	12 11 8
105	7 5 3	441	9 7 7	1078	11 7 7 2
112	8 7 2	448	8 8 7	1089	11 11 9
125	5 5 5	462	11 7 6	1125	9 5 5 5
126	9 7 2	484	11 11 4	1134	9 9 7 2
128	8 8 2	486	9 9 6	1152	12 12 8
135	9 5 3	495	11 9 5	1155	11 7 5 3
147	7 7 3	504	9 8 7	1176	8 7 7 3
154	11 7 2	512	8 8 8	1188	12 11 9
162	9 9 2	525	7 5 5 3	1215	9 9 5 3
165	11 5 3	528	12 11 4	1225	7 7 5 5
168	8 7 3	539	11 7 7	1232	11 8 7 2
175	7 5 5	567	9 9 7	1296	12 12 9
176	11 8 2	576	12 12 4	1323	9 7 7 3
189	9 7 3	588	12 7 7	1331	11 11 11
192	12 8 2	594	11 9 6	1344	8 8 7 3
196	7 7 4	605	11 11 5	1372	7 7 7 4
198	11 9 2	616	11 8 7	1375	11 5 5 5
216	12 9 2	625	5 5 5 5	1386	11 9 7 2
224	8 7 4	648	9 9 8	1408	11 8 8 2
225	9 5 5	672	12 8 7	1452	12 11 11
231	11 7 3	675	9 5 5 3	1458	9 9 9 2
242	11 11 2	686	7 7 7 2	1485	11 9 5 3
243	9 9 3	693	11 9 7	1512	9 8 7 3
245	7 7 5	704	11 8 8	1536	8 8 8 3
252	12 7 3	726	11 11 6	1568	8 7 7 4
256	8 8 4	729	9 9 9	1575	9 7 5 5
264	11 6 4	735	7 7 5 3	1584	12 12 11
275	11 5 5	756	12 9 7	1617	11 7 7 3
288	12 12 2	768	12 8 8	1694	11 11 7 2
294	7 7 6	784	8 7 7 2	1701	9 9 7 3
297	11 9 3	792	12 11 6	1715	7 7 7 5
308	11 7 4	825	11 5 5 3	1728	12 12 12
315	9 7 5	847	11 11 7	1764	9 7 7 4
324	9 9 4	864	12 9 8	1782	11 9 9 2
336	12 7 4	875	7 5 5 5	1792	8 8 7 4
343	7 7 7	882	9 7 7 2	1815	11 11 5 3
352	11 8 4	891	11 9 9	1848	11 8 7 3
363	11 11 3	896	8 8 7 2	1875	5 5 5 5 3
375	5 5 5 3	924	12 11 7	1925	11 7 5 5
378	9 7 6	945	9 7 5 3	1936	11 11 4 4
384	8 8 6	968	11 11 8	1944	9 9 8 3
385	11 7 5	972	12 9 9	2016	9 8 7 4
392	8 7 7	1008	12 12 7	2025	9 9 5 5
396	11 9 4	1024	8 8 8 2	2048	8 8 8 4

Table of Composite Numbers and their Factors—*continued.*

No.	Factors.	No.	Factors.	No.	Factors.
2058	7 7 7 6	3872	11 11 8 4	6468	12 11 7 7
2079	11 9 7 3	3888	9 9 8 6	6534	11 11 9 6
2112	11 8 8 3	3969	9 9 7 7	6561	9 9 9 9
2156	11 7 7 4	3993	11 11 11 3	6615	9 7 7 5 3
2178	11 11 9 2	4032	9 8 8 7	6655	11 11 11 5
2187	9 9 9 3	4096	8 8 8 8	6776	11 11 8 7
2205	9 7 7 5	4116	12 7 7 7	6804	12 9 9 7
2268	9 9 7 4	4125	11 5 5 5 3	6860	10 7 7 7 2
2304	9 8 8 4	4158	11 9 7 6	6875	11 5 5 5 5
2352	8 7 7 6	4224	11 8 8 6	6912	12 9 8 8
2376	11 9 8 3	4235	11 11 7 5	7056	9 8 7 7 2
2401	7 7 7 7	4312	11 8 7 7	7128	11 9 9 8
2464	11 8 7 4	4356	11 11 9 4	7168	8 8 8 7 2
2475	11 9 5 5	4374	9 9 9 6	7203	7 7 7 7 3
2541	11 11 7 3	4375	7 5 5 5 5	7392	12 11 8 7
2592	9 9 8 4	4455	11 9 9 5	7425	11 9 5 5 3
2625	7 5 5 5 3	4536	9 9 8 7	7546	11 7 7 7 2
2646	9 7 7 6	4608	9 8 8 8	7560	12 10 9 7
2662	11 11 11 2	4704	12 8 7 7	7623	11 11 9 7
2673	11 9 9 3	4725	9 7 5 5 3	7744	11 11 8 8
2688	8 8 7 6	4752	11 9 8 6	7776	12 9 9 8
2695	11 7 7 5	4802	7 7 7 7 2	7875	9 7 5 5 5
2744	8 7 7 7	4851	11 9 7 7	7938	9 9 7 7 2
2772	11 9 7 4	4928	11 8 8 7	7986	11 11 11 6
2816	11 8 8 4	5082	11 11 7 6	8019	11 9 9 9
2835	9 9 7 5	5103	9 9 9 7	8064	9 8 8 7 2
2904	11 11 8 3	5145	7 7 7 5 3	8085	11 7 7 5 3
2916	9 9 9 4	5184	9 9 8 8	8192	8 8 8 4 4
3024	9 8 7 6	5292	12 9 7 7	8232	8 7 7 7 3
3025	11 11 5 5	5324	11 11 11 4	8316	12 11 9 7
3072	8 8 8 6	5346	11 9 9 6	8448	12 11 8 8
3087	9 7 7 7	5376	12 8 8 7	8505	9 9 7 5 3
3125	5 5 5 5 5	5445	11 11 9 5	8575	7 7 7 5 5
3136	8 8 7 7	5488	8 7 7 7 2	8624	11 8 7 7 2
3168	11 9 8 4	5544	11 9 8 7	8712	11 11 9 8
3234	11 7 7 6	5625	9 5 5 5 5	8748	12 9 9 9
3267	11 11 9 3	5632	11 8 8 8	9072	9 9 8 7 2
3375	9 5 5 5 3	5775	11 7 5 5 3	9075	11 11 5 5 3
3388	11 11 7 4	5808	11 11 8 6	9216	9 8 8 4 4
3402	9 9 7 6	5832	9 9 9 8	9261	9 7 7 7 3
3456	9 8 8 6	5929	11 11 7 7	9317	11 11 11 7
3465	11 9 7 5	6048	12 9 8 7	9375	5 5 5 5 5 3
3528	9 8 7 7	6075	9 9 5 5 3	9408	8 8 7 7 3
3564	11 9 9 4	6125	7 7 5 5 5	9504	12 11 9 8
3584	8 8 8 7	6144	12 8 8 8	9604	7 7 7 7 4
3645	9 9 9 5	6174	9 7 7 7 2	9625	11 7 5 5 5
3675	7 7 5 5 3	6237	11 9 9 7	9702	11 9 7 7 2
3696	11 8 7 6	6272	8 8 7 7 2	9801	11 11 9 9
3773	11 7 7 7	6336	11 9 8 8	9856	11 8 7 4 4

Case 1.

When the multiplier is a composite number, in which no factor greater than 12 *necessarily enters.*

Rule.—Multiply the quantity by one of the factors (none of them exceeding 12) of the multiplier, the product by another, this result by another, and so on, till all the factors have been used.

Examples.

1\. 72 cwt. at 6*s.* 7¾*d.* per cwt.
72 = 12 × 6.

	s.	*d.*
	6	7¾
		12
	79	9
		6
2,0)	47,8	6

Ans. £23 18*s.* 6*d.*

Since 12 times any number of pence is obviously that number of shillings, 12 times 7¾ pence is 7¾ shillings; that is to say, 7*s.* 9*d.*; hence the result of the first multiplication is 79*s.* 9*d.*

Again, 6 times 9*d.* are 9 sixpences = 4*s.* 6*d.*: hence the final result is 478*s.* 6*d.*, or £23 18*s.* 6*d.*

2\. 96 yards at 1*s.* 10¾*d.* per yard.
96 = 12 × 8.

	s.	*d.*
	1	10¾
		12
	22	9
		8
2,0)	18,2	0

Ans. £9 2*s.* 0*d.*

3\. 42 cwt. at £4 10*s.* 7*d.* per cwt.
42 = 7 × 6

£	*s.*	*d.*
4	10	7
		7
31	14	1
		6

Ans. £190 4*s.* 6*d.*

4\. Multiply 8 lb. 5 oz. 17 dwt. 4 gr. troy by 28 = 7 × 4.

lbs.	oz.	dwt.	gr.
8	5	17	4
			7
59	5	0	4
			4

Ans. 237 lb. 8 oz. 0 dwt. 16 gr.

5. Multiply 17 lb. 7 oz. 9 dr. avoirdupois, by 168.

By a reference to the foregoing table, we find that $168 = 8 \times 7 \times 3$: we may therefore multiply by these factors in succession.

lb.	oz.	dr.
17	7	9
		8
139	12	8
		7
978	7	8
		3

Ans. 2935 lb. 6 oz. 8 dr. = 2935 lb. 6½ oz.

6. Multiply £19 13*s.* 5¼*d.* by 28. *Ans.* £550 16*s.* 3*d.*
7. Required the price of 16 cwt. of tallow at £1 18*s.* 8*d.* per cwt. *Ans.* £30 18*s.* 8*d.*
8. At 1*s.* 10¾*d.* per lb., what will 96 lb. cost? *Ans.* £9 2*s.*
9. Multiply 23 miles 1 furlong 31 perches 2 yards by 256. *Ans.* 6025 m. 6 fur. 4 per. 3 yd.
10. Multiply 7*s.* 10¾*d.* by 7986. *Ans.* £3152 16*s.* 1½*d.*

Case 2.

When the multiplier is not a composite number; or, being a composite number, when any of its factors are inconveniently large.

[The number 69, for instance, is a composite number, since it has integral factors, namely, 23 and 3; but 23 is a multiplier which is inconvenient for compound multiplication. The present case includes all numbers in each of which an integral factor higher than the number 12 necessarily enters; as also, of course, every number which has no integral factors at all, except the number itself and unit, that is, every prime number.]

Rule.—1. Refer to the table for the composite number which is the nearest in value to the given multiplier, and use, as in the last case, the factors of it.

2. Multiply the quantity by the difference between the given number and that taken from the table; and if the tabular number be less than the given one, add the product; if it be greater, subtract the product.

Examples.

1. 79 yards at 7*s.* 10*d.* per yard. Here the nearest composite number to 79 is 75 = 5 × 5 × 3, so that we work thus:

	£	*s.*	*d.*
		7	10
			5
	1	19	2
			5
	9	15	10
			3
	29	7	6
Add	1	11	4 for 4 yds. at 7*s.* 10*d.*
Ans.	£30	18*s.*	10*d.*

2. Multiply 13 acres 3 roods 17 poles, by 511. The nearest composite number is 512 = 8 × 8 × 8.

	acres.	roods.	poles.
	13	3	17
			8
	110	3	16
			8
	886	3	8
			8
	7094	1	24
Sub.	13	3	17 for 1
Ans.	7080 ac.	2 rd.	7 po.

3. What is the price of 114 stone of meat at 15*s.* 3¾*d.* per stone? *Ans.* £87 5*s.* 7½*d.*
4. What sum of money must be divided among 108 men, so that each may receive £14 6*s.* 8¼*d.*? *Ans.* £1548 4*s.* 6*d.*
5. Multiply 2 sq. yd. 8 ft. 123 in. by 563. *Ans.* 1679 sq. yd. 7 ft. 129 in.

When the given multiplier has ¼, ½, or ¾ connected with it, disregard the fraction, and proceed as above; then for ¼ add a fourth part of the quantity multiplied to the result, for ½ add half that quantity, and for ¾ add half and the half of that half; or if the multiplier actually used exceed that given by a fraction, subtract. (See Example 8.)

6. 117¼ gallons at 12*s.* 6*d.* per gallon.

117 = 112 + 5, and 112 = 8 × 7 × 2.

	£	*s.*	*d.*
		12	6
			8
	5	0	0
			7
	35	0	0
			2
	70	0	0
	3	2	6 for 5 gallons, at 12*s.* 6*d.*
		3	1½ „ ¼ „ „
Ans.	£73	5*s.*	7½*d.*

7. $85\frac{3}{4}$ cwt. at £1 7*s*. 8*d*. per cwt.

$85 = 84 + 1$, and $84 = 12 \times 7$.

	£	s.	d.	
	1	7	8	
			12	
	16	12	0	
			7	
	116	4	0	
	1	7	8	for 1 cwt.
		13	10	for $\frac{1}{2}$ } = $\frac{3}{4}$ cwt.
		6	11	for $\frac{1}{4}$ }
Ans.	£118	12*s*.	5*d*.	

8. What is the price of $87\frac{3}{4}$ bushels of wheat at 4*s*. 3*d*. per bushel?

$87 = 88 - 1$, and $88 = 11 \times 8$.

	£	s.	d.	
	0	4	3	
			11	
	2	6	9	
			8	
	18	14	0	
Sub.		4	3	for 1 bushel.
	18	9	9	
		2	$1\frac{1}{2}$	} for $\frac{3}{4}$ bush.
		1	$0\frac{3}{4}$	}
Ans.	£18	12*s*.	$11\frac{1}{4}$*d*.	

Or better thus; since $87\frac{3}{4} = 88 - \frac{1}{4}$.

	£	s.	d.	
	0	4	3	
			11	
	2	6	9	
			8	
	18	14	0	
Sub.		1	$0\frac{3}{4}$	for $\frac{1}{4}$ bush.
Ans.	£18	12*s*.	$11\frac{1}{4}$*d*.	

9. Multiply 16° 51′ 43″ by $231\frac{3}{4}$. *Ans.* 3907° 45′ $20\frac{1}{4}$″.

10. Multiply 19 days 13 hours 27 minutes by $443\frac{1}{2}$. *Ans.* 8670 d. 3 h. 42 m. 45 s.

11. Multiply 9 oz. 17 dwt. 20 gr. by $616\frac{1}{4}$. *Ans.* 6095 oz. 14 dwt. 19 gr.

12. Multiply 14 tons 13 cwt. 2 qr. 11 lb. by $243\frac{3}{4}$. *Ans.* 3578 tons 4 cwt. 2 qr. $7\frac{1}{4}$ lb.

Remarks on the real character of Multiplication, and on some erroneous opinions respecting it.

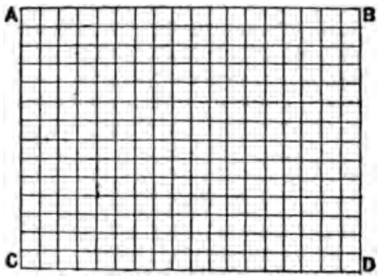
Multiplication, whether simple or compound, is a short way of finding the result of addition, and it is nothing more than this. The multiplier always expresses the number of things, each equal to the multiplicand, that are to be incorporated into one sum; and this sum, by the operation of multiplication, is furnished by the product. The multiplier, therefore, can never be anything but an abstract number; it can never be a commodity, as a sum of money or a weight of goods; nor yet any measure of length, surface, or capacity; it simply denotes how many repetitions, or times, some other abstract number, or concrete quantity, is to be taken and exhibited in one whole. There is no such thing as multiplication except by an abstract number: a multiplier always expresses a number of times, and never concrete things; the number of times, namely, that a proposed quantity, be it concrete or not, is to be taken, and all these repetitions of the same individual thing (or equivalents of that thing) incorporated in one whole; and this whole must, of course, necessarily be the same in kind as each of the individual quantities, themselves all of one kind, of which it is composed. When the multiplier is a whole number and a fraction, it directs us to take the multiplicand that whole number of times, and also to add in the proposed fractional part of a time. The multiplying a quantity by a number, whole or mixed, or fractional, means the taking that quantity the proposed number of times, together with the proposed fractional part of it, and finding the whole sum without the trouble of writing down these items and then adding them all up.

As to the apparent practical departure from the above-mentioned general principle in the particular instances of computing surfaces and solids from their given linear dimensions, we shall show, in the next article, that the departure is only apparent; and that the rules for such computations are worded as they are solely for brevity of expression, and as a sort of artificial aid to the memory of the computer, as to the successive steps of the mere figure-work.

Cross Multiplication.

At page 23 we have given a table of superficial or square measure, as it is sometimes called, the inches, feet, &c., in that table being exclusively square inches, feet, &c. By a square foot, or inch, is meant the surface, or area, of a square, each side of which is a linear foot, or a linear inch; so that when a surface is said to measure 10 square feet, the meaning is that it is equal in area to 10 times the surface of 1 square foot.

The simplest kind of plane figure is the rectangle, a figure of four sides, each pair of opposite sides being equal and parallel straight lines. The square is itself a rect-



angle; but in this particular kind of rectangle, not only are the opposite sides equal sides, but the sides are all four of them equal. A general rule for computing the area of a rectangle, from knowing the measures of the length and breadth of it, may be deduced thus:—

Suppose the length of the proposed rectangle to be 18 feet, and its breadth 14 feet; and let it be represented by the above figure, A B C D. Let the length A B be divided into 18 equal parts, and the breadth A C into 14; then if parallel lines be drawn from the points of division, as here represented, the rectangle will be cut up into squares of which each side measures one foot. By counting these squares, we find that the rectangle contains 252

square feet; but since we know that there are fourteen horizontal rows of squares between A B and C D, and that each row consists of eighteen squares, the exact number of squares will be found by simply multiplying 18 by 14; hence $18 \times 14 = 252 =$ the number of square feet in the surface. Here the 18, the 14, and the 252, are abstract numbers: we have multiplied the number of feet in the length by the number of feet in the breadth, and we know, from the above diagram, that the result is the number of square feet in the rectangular surface; and it is plain that the area would be found, in a similar manner, if the length and breadth had each been any other number of feet. And if a fraction of a foot had been added to this number of feet, the operation would still have been that of ordinary multiplication: thus, in the foregoing example, if the length had been $18\frac{1}{2}$ feet, then a vertical row of half-squares would have been added to the vertical row B D of fourteen whole squares, and each of the fourteen horizontal rows would have consisted of $18\frac{1}{2}$ square feet, so that the number of square feet in the rectangle would have been 14 times $18\frac{1}{2}$, or 259. If with this length, the breadth had been $14\frac{1}{4}$ feet, then a row of quarter-squares would have been added to the horizontal row C D of eighteen whole squares, together with a quarter of the half-square already added on at D, since a vertical column of half-squares has been added on at B D; hence the entire number of squares is 14 times $18\frac{1}{2}$ together with $\frac{1}{4}$ of $18\frac{1}{2}$; that is, the number of square feet is $18\frac{1}{2} \times 14\frac{1}{4} = 263\frac{5}{8}$, which is thus got, as before, by simply multiplying the number of feet in the length by the number of feet in the breadth.

From these particular illustrations it is obvious that, whatever be the length and breadth of a rectangle in feet and fractions of a foot, the number of square feet in the area of it is found by multiplying the number of linear feet in the one dimension by the number of linear feet in the other; and that the numbers thus multiplied together are abstract, not concrete numbers; that is, we do not multiply feet by feet.

When, however, the linear dimensions involve fractions of a foot, it is usually the more convenient course to convert

the fractions into decimals, for the work; thus, writing 18·5 for $18\frac{1}{2}$, and 14·25 for $14\frac{1}{4}$, we should multiply as in the margin. The decimal ·625 Ft. may be converted into inches by multiplying it by 144, the number of square inches in a square foot, or by multiplying by 12 and 12. The result is 90 square inches, so that the area of the rectangle is 263 Ft. 90 In. We write the initial letters of the feet and inches in capitals, in order to distinguish them from linear feet and inches; this form should, we think, be generally used, to save the trouble of always inserting the word "square."

$$\begin{array}{r} 14{\cdot}25 \\ 18{\cdot}5 \\ \hline 7125 \\ 25650 \\ \hline 263{\cdot}625 \text{ Ft.} \\ \hline \end{array}$$

But there is another way of calculating the area, usually practised by workmen, when the linear dimensions are taken in feet and inches. This is by cross multiplication.

Take the instance already considered, the length of the rectangle being 18 ft. 6 in., and its breadth 14 ft. 3 in., or, as it is usually expressed, the dimensions being 18 ft. 6 in., by 14 ft. 3 in. Now as in ordinary multiplication we carry the number of tens, so here we must carry the number of twelves. In $6 \times 3 = 18$, there is 1 twelve, therefore, to be carried, and the overplus 6 to be put down; in 18×3, *plus* the 1 carried, that is, in 55, there are 4 twelves, and 7 over. The first line of the multiplication is therefore to be written as in the margin; it expresses $4 + \frac{7}{12} + \frac{6}{144}$, just as 4·76 expresses $4 + \frac{7}{10} + \frac{6}{100}$. The 7, in the above work, denotes seven-twelfths of one of the units in the 4, and the 6 denotes six-twelfths of one of the units in the 7, or six 144ths of one of the units in the 4.

$$\begin{array}{rrr} 18 & 6 & \\ 14 & 3 & \\ \hline 4 & 7 & 6 \\ 259 & 0 & \\ \hline 263 & 7 & 6 \\ \hline \end{array}$$

For the second line of the work we have $6 \times 14 = 84 =$ seven twelves, and nothing over; and this seven added to 18×14 gives 259. The product is therefore 263 7 6, so that the area of the rectangle is 263 square feet, 7 twelfths of a square foot, and 6 square inches. The twelfth of a square foot is 12 square inches, therefore the area is 263 Ft. 90 In., or, as at page 39, $263\frac{5}{8}$ Ft.

It is customary to call the twelfth of a square foot a *Part*, certainly a very vague designation. Using this term, however, the foregoing result would be read, 263 Ft. *7 Pts. 6 In.* It would be far better, however, to call these

Parts twelfths, meaning twelfths of a square foot, each "Part" being a surface one foot in length and one inch in breadth; these Parts, or twelfths, are, however, usually turned into Inches, and the result expressed in Feet and Inches, and fractions of Inches only.

It should here be mentioned that in the numerical operation above, we have arranged the steps of the work just as they would be arranged if we had been dealing with ordinary decimals instead of with duodecimals, in order that the strict analogy between the two processes may be clearly seen. But in actual practice this conformity of mere arrangement is usually departed from, the second line of the work being placed first, agreeably to the following:—

Rule *for computing the surface of a rectangle.* Set down the two dimensions one under the other, feet under feet, and inches under inches, and work as follows:—

1. Commence with the feet in the multiplier, and multiply each term of the multiplicand, beginning with the inches (or lowest denomination), by that number. For every 12 in the product carry 1; and write the overplus, or remainder, under that term of the multiplicand which has supplied the product.

2. In like manner multiply the multiplicand, beginning as before with the lowest denomination in it, by the number of inches in the multiplier, rejecting, as before, the twelves in the product (carrying 1 for each 12), and here write the remainder one place further to the right of the term multiplied. Add the two lines together, and the result will be the area in Feet, Parts, and Inches.

Should the given dimensions consist of more than two duodecimal denominations (feet and inches), and include twelfths of inches, and even twelfths of one of these twelfths, the same course is to be followed. At each multiplication the term first written down is to be removed one place more to the right, since a unit of each multiplier (after the feet) is always one-twelfth of a unit of the immediately preceding multiplier, and the denominations, from left to right, uniformly descend by twelfths. See the third working of example 9, at page 43.

Note.—For the more easy recollection of the denominations to which each partial result in the several steps of the work refers, it is common

to say (and for this purpose only, allowable to say) that feet multiplied by feet produce Feet; inches multiplied by inches produce Inches; and inches multiplied by feet produce Parts, or (which is a better term) twelfths of a Foot.

We shall now give a few examples, commencing with the example already worked, under the slightly different arrangement alluded to above.

EXAMPLES.

	feet.	inches.	
1.	18	6	
	14	3	
	259	0	
	4	7	6
	263 Ft.	7 Pts.	6 In.

	feet.	inches.	
2.	17	9	
	20	6	
	355	0	
	8	10	6
	363 Ft.	10 Pts.	6 In.

	feet.	inches.	
3.	22	5	
	16	11	
	358	8	
	20	6	7
	379 Ft.	2 Pts.	7 In.

And, turning the Parts into Inches, these several results are:
263 Ft. 90 In., 363 Ft. 126 In., and 379 Ft. 31 In.

4. Find the area of a rectangle, the dimensions being 8 ft. 5 in. by 4 ft. 7 in. *Ans.* 38 Ft. 83 In.
5. Required the area of a rectangle 9 ft. 8 in. by 7 ft. 6 in. *Ans.* 72 Ft. 72 In.
6. What is the area of a rectangle 75 ft. 7 in. by 9 ft. 8 in.? *Ans.* 730 Ft. 92 In.
7. Find the area when the dimensions are 75 ft. 9 in. by 17 ft. 7 in. *Ans.* 1331 Ft. 135 In.
8. Find the area when the dimensions are 179 ft. 3 in. by 38 ft. 10 in. *Ans.* 6960 Ft. 126 In.
9. Required the area of a plank 20¾ feet long and 12½ inches broad. *Ans.* 21 Ft. 88½ In.

As fractions occur in these dimensions, we shall give the work first by cross multiplication and then by the ordinary methods.

feet.	in.	
20	9	
1	0½	
20	9	
	10	4½
21 Ft.	7 Pts.	4½ In.
= 21 Ft.	88½ In.	

feet. in. in.
20 9 = 249
12½
2988
124½
12) 3112½ In.
12) 259 Pts. 4½ In.
21 Ft. 7 Pts. 4½ In.

The first method is evidently the shorter and the more convenient; but the following mode of working (the $\frac{1}{2}$ being replaced by six twelfths), is upon the whole to be preferred. We shall indicate the twelfth of a linear inch by a dash (′).

feet.	in.	′		
20	9	0		
1	0	6		
20	9	0		
	10	4	6	0

(See the RULE.)

21 Ft. 7 Pts. 4 In. 6‵ = 21 Ft. $88\frac{1}{2}$ In.

Among mechanics, generally, linear measurements are not taken to a greater nicety than the 16th of an inch, their divisions of the inch being the half, the quarter, the eighth, and the sixteenth. In cross multiplication these are all to be replaced by their equivalents in twelfths, and this is easily done, for

$$\frac{1}{2}=\frac{6}{12};\ \frac{1}{4}=\frac{3}{12};\ \frac{1}{8}=\frac{3}{24}=\frac{1\frac{1}{2}}{12};\ \frac{1}{16}=\frac{3}{48}=\frac{\frac{3}{4}}{12};$$

that is, using, as above, the simple dash to mark twelfths of a linear inch, the double dash to mark twelfths of these twelfths, and so on, we have

$$\overset{\text{in.}}{\frac{1}{2}}=6';\ \overset{\text{in.}}{\frac{1}{4}}=3';\ \overset{\text{in.}}{\frac{1}{8}}=1'\,6'';\ \overset{\text{in.}}{\frac{1}{16}}=\frac{3}{4}\text{ of }1'=\frac{3}{4}\text{ of }12''=9''.$$

10. Find by duodecimals the area of a rectangle measuring 7 ft. $5\frac{3}{4}$ in. by 3 ft. $5\frac{1}{4}$ in. *Ans.* 25 Ft. 8 Pts. 6 In. 2‵ 3‶; or 25 Ft. $102\frac{3}{16}$ Ins.

It may seem that we have worked this last example to an unnecessary degree of precision. A practical man would reject from his result the small portions of surface marked 2‵ and 3‶, the reversed dashes being used here to distinguish these portions of surface from the linear measures 2′ and 3″. Yet, that the result may be correct to the nearest square inch, he must first compute it to this extent, rejecting the insignificant parts of it afterwards.

When the highest denomination in the linear measure-

ment exceeds feet, as yards, poles, &c., this higher denomination must be reduced to feet. The result being obtained in square feet, &c., it may then be converted into square yards, &c., as in the following example.

11. How many square yards are there in a carpet 7 yd. 1 ft. 4 in. long and 5 yd. 2 ft. 3 in. wide?

feet.	in.	
22	4	
17	3	
379	8	
5	7	0

9) 385 Ft. 3 Pts. = 42 Yd. 7 Ft. 36 In. = 42 Yd. $7\frac{1}{4}$ Ft.

12. What is the area of a rectangle 13 yd. 2 ft. 9 in. by 5 yd. 1 ft. 7 in.? *Ans.* 76 Yd. 8 Ft. 51 In.

COMPOUND DIVISION.

There are two distinct aspects under which division should be viewed. When we are required to divide a concrete quantity by an abstract number, as, for instance, by 4, 9, 24, &c., the demand is to determine the 4th, 9th, 24th, &c., part of that quantity; but when we are required to divide one concrete quantity by another concrete quantity of the same kind, then the demand is to find how many times the divisor is contained in the dividend; and the quotient, or answer to the inquiry, is then always an abstract number. If it be required to divide £34 16*s.* 8*d.* by 24, the answer (to the nearest farthing) is £1 9*s.* $0\frac{1}{4}$*d.*, this being the 24th part of the given sum; but if we had been required to divide by 24*s.*, that is, to find how many sums, each equal to £1 4*s.*, are contained in £34 16*s.* 8*d.*, the answer would have been 29 of such sums with 8*d.* over; in other words, that £34 16*s.* 8*d.* contains 24*s.* 29 times with 8*d.* to spare.

In order that he may have correct notions of what he is about, the reader should be careful to discriminate between these two kinds of division; to notice that when he is

required to divide a concrete quantity by a mere number, the quantity is supposed to be cut up into that number of equal parts, and his business is to find the value of one of those parts; and that when he is required to divide one concrete quantity by another, then he is to find how many times the one quantity is contained in the other: he does not here get a concrete quantity for his result, as in the former case, but an abstract number.

From these observations, the reader will at once see that however repugnant to common sense it may be (and pure nonsense it certainly is) to speak of multiplying money by money, yet we may propose to divide money by money, that is, to find how many times a smaller sum is contained in a greater, with strict propriety. If money be the multiplicand, the multiplier must be a mere number; if money be the dividend, the divisor may be either a mere number, or be itself money: in the former case the quotient will be money, in the latter it will be an abstract number.

We shall now give the necessary rules for these two cases, illustrating them by suitable examples.

Case I.

When the divisor is an abstract number.

Rule.—Apply the divisor to the leading, or highest, denomination in the compound quantity, and if the divisor do not exceed 12, put the quotient underneath; reduce the remainder to the next lower denomination, and add it, thus reduced, to the term of that lower denomination in the dividend. Divide the sum in the same way, reducing the remainder to the next lower denomination, and adding in the term of like denomination in the dividend, as before; and proceed in this way from term to term to the lowest denomination.

And if the divisor be greater than 12, but be composed of factors none of which exceed 12, we may still proceed by short division as above directed, with each of these factors in succession, instead of dividing at once by the composite number itself. Whether the proposed divisor can or cannot be decomposed into factors suitable for the

operation of short division, we may discover at once by reference to the table at page 31. But when the divisor is a large number, the factors of which are unsuitable for these successive short division steps, reduce the compound quantity to the lowest denomination in it, and *then* divide. The quotient will be a quantity in that lowest denomination, which may be brought into the higher denomination by reduction.

Note.—When the number in the highest denomination actually contains the divisor, we may apply the divisor to it at once, and reduce what remains to the lowest denomination afterwards. (See Ex. 3.)

Examples.

1. Find the 24th part of £34 16s. 8d.

$$24 = 8 \times 3$$

	£	s.	d.
8)	34	16	8
3)	4	7	1
	£1	9s.	$0\frac{1}{4}d. + \frac{1}{3}far.$

2. Divide £780 12s. 4d. by 168.

By the table (p. 31), $168 = 7 \times 6 \times 4$.

	£	s.	d.
4)	780	12	4
6)	195	3	1
7)	32	10	6 . . 1
	£ 4	12s.	11d. . 1

Now, as already explained (p. 11), the fraction of a penny belonging to this final quotient, is $1 \times 6 + 1$ divided by 7×6, or 42; that is, it is $\frac{7}{42}d.$, or one-sixth of a penny;* so that the sum, to be rendered accurately divisible by 168, should be diminished by 168 of these sixths, that is, since $\frac{168}{6}d. = 28d.$, it should be diminished by 2*s.* 4*d.*; or else it should be increased by 168 farthings less this sum; which increase is 1*s.* 2*d.* In the former case, the exact 168th part would be £4 12*s.* 11*d.*, and in the latter, a farthing more than this. We shall here work the example under both these changes.

* The complete quotient from the divisor 6 is £32 10*s.* 6*d.* $+ \frac{1}{6}d.$; and the next complete quotient is therefore £4 12*s.* 11*d.* $+ \frac{1}{7}d. + \frac{1}{42}d.$; and the sum of these two fractions is $\frac{1 \times 6}{42} + \frac{1}{42} = \frac{7}{42}$.

	£	s.	d.			£	s	d.
4)	780	10	0		4)	780	13	6
6)	195	2	6		6)	195	3	4½
7)	32	10	5		7)	32	10	6¾
	£4	12*s*.	11*d*.			£4	12*s*.	11¼*d*.

This work merely verifies the above conclusion, reached at once with scarcely any work at all; and when any sum is proposed for division, and we find, upon trial, that accurate division, without fractions of a farthing, is impracticable, we thus see how readily we may find the smallest amount by which the proposed sum must be diminished or increased in order that the required part of it may be practically payable in existing coin.

3. £837 13*s*. 6*d*. is to be equally divided among a ship's company of 273 persons; what will be the share of each?

As we see, by a reference to the table at p. 31, that 273 is not decomposable into factors suitable for short division, we must proceed by long division, as follows:—

The lowest denomination being 6*d*., we shall reduce the sum to sixpences.

£ s. d.	£ s. d. £ s. d.
837 13 6 Or, better thus:	273) 837 13 6 (3 1 4¼
20	819
16753	18
2	20
sixpences. 273) 33507 (122 = 61*s*. = £3 1*s*.	373 (1*s*.
273	273
620	100
546	12
747	1206 (4*d*.
546	1092
201	114
6	4
1206 (4*d*.	456 (1 *far*.
1092	273
114	183
4	
456 (1 *far*.	
273	
183	

Ans. £3 1*s*. 4¼*d*. + $\frac{183}{273}$ *f*.

4. Divide £64 19*s.* by 36. *Ans.* £1 16*s.* 1*d.*
5. Divide £46 14*s.* 6*d.* by 24. *Ans.* £1 18*s* 11¼*d.*
6. Divide 315 days 17 hours 37 minutes by 112.
   Ans. 2 days 19 hours 39 minutes 26¼ seconds.
7. Divide 128° 45′ 52″ by 125. *Ans.* $1° 2' 48'' + \frac{52}{125}''$.
8. Divide 496 miles 5 furlongs 2 perches by 594.
   Ans. 6 furlongs 27 perches 3 yards.
9. Divide 7080 acres 2 roods 7 poles by 511. *Ans.* 13 ac. 3 rd. 17 po.
10. Divide 1679 sq. yd. 7 sq. ft. 129 sq. in. by 563.
    Ans. 2 Yd. 8 Ft. 123 In.

*** Why the initial letters in this last answer are capitals is explained at page 40.

Case II.

When the divisor is a concrete quantity.

Rule.—Reduce the two quantities to the lowest denomination that occurs in either of them, and then perform the division with the results: the quotient will express the number of times the smaller of the two concrete quantities is contained in the greater.

Examples.

1. How many times is £2 17*s.* 8*d.* contained in £18 3*s.*?

The lowest denomination here is pence; we have therefore to reduce both divisor and dividend to pence, and to perform the division with the results, thus:—

```
£  s.  d.                 £   s.  d.
2  17  8                  18  3   0
20                         20
—                         ——
57                        363
12                         12
—                         ——
692                 692) 4356 (6
———                      4152
                         ——
                          204 rem. = 17s. over.
                         ——
```

Hence the smaller sum is contained in the greater six times, with 204 pence to spare. If the £18 3*s.* be diminished by this, that is, by 17*s.*, it will then contain the £2 17*s.* 8*d.* exactly 6 times. As it is, however, the greater contains the less 6 times and the $\frac{204}{692}$ of that less *besides.*

2. How many parcels, each weighing 9 oz. 17 dwt. 20 gr., may be made up out of 380 lb. 10 oz. 14 dwt. 19 gr.?

```
oz. dwt. gr.                                  lb.  oz. dwt. gr.
 9   17  20                                   380  10   14  19
     20 = number of dwt. in 1 oz.                  12 = number of oz. in 1 lb.
    ---                                       ----
    197                                       4570
     24 = number of gr. in 1 dwt.               20
    ---                                      -----
    788                                      91414
   396                                          24
   ----                                     ------
   4748 gr. in each parcel.                 365665
   ----                                    182829
                                          -------
                                    4748) 2193955 (462
                                          18992
                                          -----
                                           29475
                                           28488
                                           -----
                                             9875
                                             9496
                                             ----
                                              379
```

It thus appears that the number of parcels is 462, and that there will be 379 grains to spare; hence the greater weight contains the smaller $462\frac{379}{4748}$ times.

There are no means of abridging operations of this kind.

3. Divide £46 14*s*. 6*d*. by £3 17*s*. $10\frac{1}{2}$*d*. *Ans*. 12.
4. Divide 2 tons 13 cwt. 5 lb. by 3 qr. 17 lb. *Ans*. $58\frac{83}{101}$.
5. Divide 8631 days 48 minutes 45 seconds by 19 days 13 hours 27 minutes. *Ans*. $441\frac{1}{4}$.

COMPENDIOUS METHODS OF CALCULATION IN SPECIAL CASES.

In the preceding pages we have given general rules of operation universally applicable to all inquiries coming under the several heads into which the subjects treated of have been divided; and, in a few instances, we have replaced hackneyed methods by shorter modes of proceeding. It is our purpose now to explain how, by the exercise of a little ingenuity, and by means of certain obvious expedients, these general rules may, in special cases, be put aside altogether; and particular rules, expressly applicable to such cases, and involving less numerical work, be substituted for them.

In the various avocations of civilized life, those in which calculation is essential are usually such that each distinct

calling brings into exercise one particular class of arithmetical operations much more frequently than any other class: that portion of arithmetic which might do very well for the draper and the mercer, would not suffice for the carpenter or the bricklayer; and what might fully answer the requirements of either of the latter would be insufficient for the spirit merchant or the banker: they would, indeed (viewed commercially), be quite useless to him.

We shall here give a series of special rules of calculation, for ready use in special commercial and handicraft callings; and shall in each case show the consistency between the abbreviated operation and the more lengthy process of ordinary arithmetic. By these compendious rules of computation, in special instances, numerical work may be shortened, and time and labour economized. But since certain of these compendious rules involve frequent use of fractional parts of the £ and of the shilling, we shall first give a Table of all such fractional parts as can be expressed in current coin; the numerator of each fraction being 1.

Table of Fractional Parts of £1 *and of* 1*s*.

Parts of £1.	Parts of 1*s*.	———
$\frac{1}{2}$ = 10*s*.	$\frac{1}{2}$ = 6*d*.	NOTE.—All these fractions of a
$\frac{1}{3}$ = 6*s*. 8*d*.	$\frac{1}{3}$ = 4*d*.	shilling are, of course, also frac-
$\frac{1}{4}$ = 5*s*.	$\frac{1}{4}$ = 3*d*.	tions of £1; found, in each case,
$\frac{1}{5}$ = 4*s*.		by multiplying the denominator
$\frac{1}{6}$ = 3*s*. 4*d*.	$\frac{1}{6}$ = 2*d*.	by 20. We thus have:—
$\frac{1}{8}$ = 2*s*. 6*d*.	$\frac{1}{8}$ = $1\frac{1}{2}$*d*.	£$\frac{1}{160}$ = $1\frac{1}{2}$*d*.
$\frac{1}{10}$ = 2*s*.		
$\frac{1}{12}$ = 1*s*. 8*d*.	$\frac{1}{12}$ = 1*d*.	
$\frac{1}{15}$ = 1*s*. 4*d*.		
$\frac{1}{16}$ = 1*s*. 3*d*.	$\frac{1}{16}$ = $\frac{3}{4}$*d*.	£$\frac{1}{320}$ = $\frac{3}{4}$*d*.
$\frac{1}{20}$ = 1*s*.		
$\frac{1}{24}$ = 10*d*.	$\frac{1}{24}$ = $\frac{1}{2}$*d*.	£$\frac{1}{480}$ = $\frac{1}{2}$*d*.
$\frac{1}{30}$ = 8*d*.		
$\frac{1}{32}$ = $7\frac{1}{2}$*d*.		
$\frac{1}{40}$ = 6*d*.		
$\frac{1}{48}$ = 5*d*.	$\frac{1}{48}$ = $\frac{1}{4}$*d*.	£$\frac{1}{960}$ = $\frac{1}{4}$*d*.
$\frac{1}{60}$ = 4*d*.		
$\frac{1}{64}$ = $3\frac{3}{4}$*d*.		
$\frac{1}{80}$ = 3*d*.		*** We thus see that the shil-
$\frac{1}{96}$ = $2\frac{1}{2}$*d*.		ling (48 farthings) has nine
$\frac{1}{120}$ = 2*d*.		different divisors or factors, and
$\frac{1}{192}$ = $1\frac{1}{4}$*d*.		that the pound (960 farthings)
$\frac{1}{240}$ = 1*d*.		has twenty-seven.

PROBLEM 1.

To find how many articles of the same kind, may be bought for a given number of pounds (without shillings or pence), when the price of one is an even number of shillings.

By the general rule at p. 45, we should bring the given pounds into shillings, in order that both sums may be of the same denomination; we should then divide the number of shillings, thus obtained, by the number of shillings in the price of one article; the quotient would be the number of articles required. To bring the pounds into shillings would require a multiplication by 20; but as, here, the divisor is an even number, we take advantage of this circumstance, and divide half the dividend by half the divisor; that is, we work by the following special rule, observing that a number is multiplied by 10 by simply annexing to it a cipher.

RULE.—Annex a cipher to the given number of pounds, and then divide by half the given number of shillings.

EXAMPLES.

1. If a yard of cloth cost 8*s*., how many yards may be bought for £16? $160 \div 4 = 40$ yards.
2. How many yards, at 6*s*. per yard, can be bought for £48? $480 \div 3 = 160$ yards.
3. How much sugar, at 30*s*. per cwt., can be bought for £80?

$$\begin{array}{r|l} 15) & 800 \\ \hline & 53\tfrac{1}{3} \text{ cwt.} = 53 \text{ cwt. } 1 \text{ qr. } 9\tfrac{1}{3} \text{ lb.} \end{array}$$

Here the fraction which completes the quotient is $\frac{5}{15}$, which, in its lowest terms, is $\frac{1}{3}$; and $\frac{1}{3}$ cwt. is $\frac{4}{3}$ qr. $= 1\frac{1}{3}$ qr. $= 1$ qr. $+ \frac{28}{3}$lb. $= 1$ qr. $9\frac{1}{3}$ lb. And this little additional work may be readily executed mentally. In the following example, however, the use of the pen will be found necessary.

4. If 1 cwt. cost 26*s*., how much can be bought for £80?

The work, at length, is as follows.

```
13) 800
    ———
    61 cwt., 7 remainder.
              4
             ——
     13) 28 qr.
             ——
             2 qr., 2 rem.
                   28
                   ——
            13) 56 lb.
                ——
                4 lb., 4 rem.
                      16
                      ——
               13) 64 oz.
                   ——
                   4 oz., 12 rem.
                          16
                          ——
                   13) 192 dr.
                       ————
                       14 10/13.
                       ————
```

Ans. 61 cwt. 2 qr. 4 lb. 4 oz. $14\frac{10}{13}$ dr.

5. How many cwt. of butter, at 42*s.* per cwt., can be bought for £126? *Ans.* 60 cwt.
6. How many tons of coals, at 24*s.* a ton, can be bought for £25? *Ans.* 20 tons $16\frac{2}{3}$ cwt.
7. How many barrels of ale, at 70*s.* per barrel, can be bought for £27? *Ans.* $7\frac{5}{7}$, or 7 bar. $25\frac{5}{7}$ gal.
8. How many gallons of brandy, at 32*s.* per gallon, can be bought for £34? *Ans.* $21\frac{1}{4}$ gal.

NOTE.—In some of the examples to which the foregoing rule is applicable, a mode of proceeding even more easy and convenient may be employed: we refer to those instances in which the price, in shillings, of the single article, ends with a cipher; as 30*s.*, 50*s.*, 70*s.*, &c. Thus, taking example 3, where the price of 1 cwt. is 30*s.*, this price in pounds is £$\frac{30}{20}$ = £$\frac{3}{2}$: therefore $\frac{3}{2}$ becomes the divisor of the 80: but since the quotient remains the same, by whatever number both dividend and divisor are multiplied, we may here take the double of both; that is, 160 for dividend, and 3 for divisor; when the work will be simply this—

```
3) 160
   ———
   53 1/3 cwt.
```

In like manner if the price had been 70*s.*, then instead of dividing 800 by 35, we may divide twice 80 by 7 (which is easier) and we shall get the same result. Both methods are here exhibited.

```
35) 800 (22 30/35 = 22 6/7; or 7) 160
    70                           ———
    ——                           22 6/7
    100                          ———
     70
    ——
     30
```

And similarly in all such cases; the general precept being this:— Suppress the final 0 in the price, and divide twice the given number of pounds by what is left. For, expressing the price in pounds,—

$$30s. = £\frac{30}{20} = £\frac{3}{2};\ 50s. = £\frac{50}{20} = £\frac{5}{2};\ 70s. = £\frac{70}{20} = £\frac{7}{2};$$
$$90s. = £\frac{90}{20} = £\frac{9}{2};$$

and so on, for 110*s*., 130*s*., &c.; the multiplier for the given number of pounds being always 2, and the divisor the number of shillings, when the final 0 in that number is expunged. Of course, we do not notice here the cases in which the price is 20*s*., 40*s*., 60*s*., &c., because as these sums express £1, £2, £3, &c., respectively, we should merely have to divide the given number of pounds by the number which denotes the price of the single article.

There are cases, too, in which the number of shillings in the price is odd, which deserve special mention; the cases, namely, in which that number terminates with a 5; as 15, 25, 35, 45, &c. Every such number is divisible by 5; and since 20 is also divisible by 5, we have—

$$15s. = £\frac{15}{20} = £\frac{3}{4};\ 25s. = £\frac{25}{20} = £\frac{5}{4};\ 35s. = £\frac{35}{20} = £\frac{7}{4};\ \&c.;$$

the multiplier of the given number of pounds being always the fifth part of 20 (namely, 4), and the divisor, the fifth part of the number of shillings; it being remembered that to divide by a fraction we merely turn that fraction upside down (or reverse its terms), and multiply: thus, if we are required to find how many articles at 35*s*. each can be bought for £21, we have—

$$21 \div \frac{7}{4} = 21 \times \frac{4}{7} = 12, \text{ the number required.}$$

In like manner, if the price of one were 75*s*., the number purchasable for £21 would be—

$$21 \div \frac{15}{4} = 21 \times \frac{4}{15} = 7 \times \frac{4}{5} = \frac{28}{5} = 5\tfrac{3}{5}.$$

The general precept, in all such cases, is this, namely— Divide 4 times the number of pounds by a fifth part of the number of shillings. So that, in these two examples, all that need be put upon paper is

$$\text{1st. } \frac{21 \times 4}{7} = 12. \qquad \text{2nd. } \frac{21 \times 4}{15} = \frac{28}{5} = 5\tfrac{3}{5}.$$

And similarly in every instance in which the price of the single article, in shillings, is a number ending with the figure 5.

Problem 2.

The price of one article being given, to find the price of twelve.

It is evident that 12 articles, at any number of pence each, will cost just as much as that number of articles at 12 pence, or 1 shilling each; hence the

Rule.—Call the pence which one article costs shillings; and we shall thus have the price of twelve.

Examples.

1. If a pound of sugar cost $7d.$, what will 12 lbs. cost? *Ans.* $7s.$
2. If a yard cost $6\frac{1}{2}d.$, what will 12 yds. cost? *Ans.* $6\frac{1}{2}s. = 6s.\ 6d.$
3. If a yard cost $9\frac{1}{4}d.$, what will 12 yds. cost? *Ans.* $9\frac{1}{4}s. = 9s.\ 3d.$
4. If a yard cost $5\frac{3}{4}d.$, what will 12 yds. cost? *Ans.* $5\frac{3}{4}s. = 5s.\ 9d.$
5. If a pound cost $1s.\ 4\frac{1}{2}d.$, what will 12 lbs cost?
   $1s.\ 4\frac{1}{2}d. = 16\frac{1}{2}d.$; and $16\frac{1}{2}s. = 16s.\ 6d.$, the *Ans.*
6. If a piece of calico cost $5s.\ 3\frac{3}{4}d.$, what would 12 such pieces cost?
   $5s.\ 3\frac{3}{4}d. = 63\frac{3}{4}d.$; and $63\frac{3}{4}s. = £3\ 3s.\ 9s.$, the *Ans.*
7. What is the price of 12 oz. of silver, at $4s.\ 7\frac{3}{16}d.$ per oz.? As the price per oz., in pence, is to be regarded as so many shillings, we have to determine what $\frac{3}{16}$ of a shilling is.* Multiplying therefore the fraction by 12, we have—

$$\frac{3}{16}s. = \frac{36}{16}d. = \frac{9}{4}d. = 2\frac{1}{4}d.,\text{ so that, since}$$

$4s.\ 7d. = 55d.$; and that $55s. = £2\ 15s.$, therefore $55\frac{3}{16}s. = £2\ 15s.\ 2\frac{1}{4}d.$

Problem 3. (Converse of Prob. 2.)

The price of twelve being given in shillings, to find the price of one.

The price of one will of course be the twelfth part of the price of twelve; and since the twelfth part of every shilling is a penny, the rule is as follows:—

Rule.—As many shillings as twelve cost, so many pence will one cost.

* See, however, the Table at page 50 for $\frac{1}{16}s.$, which is 3 farthings, and therefore $\frac{3}{16}s.$ is 9 farthings, or $2\frac{1}{4}d.$

EXAMPLES.

1. If 12 pigeons cost 8*s.*, what is the price of one? *Ans.* 8*d.*
2. If 12 yds. cost 16*s.*, what is the price of 1 yd? *Ans.* 16*d.* = 1*s.* 4*d.*
3. If 12 pairs of socks cost 4*s.* 8*d.*, what will one pair cost?
   The cost of 12, in shillings, is 4⅔*s.*; hence the cost of 1 is 4⅔*d.*
4. If 12 gallons cost £1, what is the price per gallon?
   Ans. 20*d.* = 1*s.* 8*d.*
5. If a dozen pairs of gloves cost £1 7*s.*, what is that per pair?
   Ans. 27*d.* = 2*s.* 3*d.*
6. If I give £1 17*s.* 9*d.* for 12 lbs. of tea, what does it cost me per lb.?
   £1 17*s.* 9*d.* = 37¾*s.*; hence the cost per lb. is 37¾*d.* = 3*s.* 1¾*d.*

PROBLEM 4.

The price of one being given in pence, to find the price of any number which is a multiple of 12; *that is, which contains* 12 *without remainder.*

RULE.—Call the pence shillings, and then multiply by the number of twelves. For the price of 1 twelve is twelve times that of 1.

EXAMPLES.

1. What is the price of 24 yards of calico, at 3¾*d.* per yard?
   3¾*s.* = 3*s.* 9*d.*, which multiplied by 2, gives 7*s.* 6*d.*, the *Ans.*
2. What are 36 lbs. of sugar worth, at 4½*d.* per lb.?
   4½*s.* = 4*s.* 6*d.*, and three times this is 13*s.* 6*d.*, the *Ans.*
3. Required the cost of 72 lb. of meat, at 9¼*d.* per lb?
   9¼*s.* = 9*s.* 3*d.*, and six times this is £2 15*s.* 6*d.*, the *Ans.*
4. 96 door-locks, at 3*s.* 7½*d.* each?
   3*s.* 7½*d.* = 43½*d.*, and 43½*s.* = £2 3*s* 6*d.*: this × 8 = £17 8*s.*
5. 120 pairs of gloves, at 2*s.* 3½*d.* a pair?
   2*s.* 3½*d.* = 27½*d.*, and 27½*s.* × 10 = 275*s.* = £13 15*s.*
6. 120 gallons of rum, at 13*s.* 10*d.* per gallon?

$$13s.\ 10d. = 166d., \text{ and } 166s. \times \frac{10}{20} = £\frac{166}{2} = £83.$$

NOTE.—As in this example there are 10 twelves, it is obviously only necessary to divide the number of pence (regarded as so many pounds) in the price of one gallon, by 2. We might have done the same in the preceding example: thus, £27 10*s.* ÷ 2 = £13 15*s.* A general rule may often be advantageously departed from in particular examples coming under it.

7. 132 quarters of barley, at £1 13*s.* 9*d.* per quarter?
   33*s.* 9*d.* = 405*d.*, and 405*s.* × 11 = 4455*s.* = £222 15*s.*
8. 108 yards of cloth at 2*s.* 9½*d.* per yard?
   33½*s.* × 9 = 301½*s.* = £15 1*s.* 6*d.*
9. 84 yards of silk velvet, at 9*s.* 8¾*d.* per yard?
   116¾*s.* = 116*s.* 9*d.*; which, × 7, is 817*s.* 3*d.* = £40 17*s.* 3*d.*

PROBLEM 5.

The price of one being given, in pence, to find the price of any number which is not a multiple of 12.

RULE. 1.—Find the number of twelves in the proposed number, and reserve the remainder.

2. Compute for this multiple of 12, as in last problem; then multiply the price of one by the reserved remainder, and add the product to the former result. Or, increase the aforesaid multiple of 12 by a unit, and compute by it; then multiply the price of one by what the remainder wants of 12, and subtract.

EXAMPLES.

1. What is the price of $25\frac{1}{2}$ stone of wheat, at $17\frac{1}{2}d.$ per stone?

Here there are two twelves in the proposed number, with $1\frac{1}{2}$ remainder; hence, proceeding by the Rule,—

$17s.\ 6d. \times 2 =$	£1	15s.	0d.
then $17\frac{1}{2}d. \times 1\frac{1}{2} =$		1	$5\frac{1}{2}$, for the 1.
			$8\frac{3}{4}$, for the $\frac{1}{2}$.
	£1	17s.	$2\frac{1}{4}d.$, the *Ans.*

2. 137 lbs. of worsted, at $17\frac{1}{2}d.$ per lb? Here $137 \div 12 = 11$, 5 *rem.*

$17s.\ 6d. \times 11 =$	£9	12s.	6d.
and $1s.\ 5\frac{1}{2}d. \times 5 =$		7	$3\frac{1}{2}$
	£9	19s.	$9\frac{1}{2}d.$, the *Ans.*

104 yards of cloth, at $8s.\ 6\frac{3}{4}d$ per yard? $104 \div 12 = 8$, 8 *rem.*

As the remainder here is only 4 less than 12, we shall increase the multiple (8) of 12, by 1, and then employ this 4.

$8s.\ 6\frac{3}{4}d. = 102\frac{3}{4}d.$; and $102\frac{3}{4}s. =$ £5 2s. 9d.

£5 2s. 9d. $\times 9 =$	£46	4s.	9d.	
$8s.\ 6\frac{3}{4}d. \times 4 =$	1	14	3	*Subtract.*
	£44	10s.	6d., the *Ans.*	

4. $76\frac{1}{2}$ gallons of rum, at $14s.\ 8\frac{1}{2}d.$ per gallon? $76\frac{1}{2} \div 12 = 6$, $4\frac{1}{2}$ *rem.*

$14s.\ 8\frac{1}{2}d. = 176\frac{1}{2}d.$; and $176\frac{1}{2}s. =$ £8 16s. 6d.

£8 16s. 6d. $\times 6 =$	£52	19s.	0d.
$14s.\ 8\frac{1}{2}d. \times 4\frac{1}{2} =$	2	18	10 for the 4.
		7	$4\frac{1}{4}$ for the $\frac{1}{2}$.
	£56	5s.	$2\frac{1}{4}d.$, the *Ans.*

5. 90 lbs. of tobacco, at 3*s*. 6½*d*. per pound? *Ans*. £15 18*s*. 9*d*.
6. 47 cwt. of flour, at 16*s*. 8½*d*. per cwt.? *Ans*. £39 5*s*. 3½*d*.
7. 52 acres of land, at £1 3*s*. 6*d*. per acre? *Ans*. £61 2*s*.

The converse of this problem, which is—From the price of any number of articles, not a multiple of 12, to find the price of a single article,—does not admit of being worked otherwise than by the general rule. But when the given number is a multiple of 12, the following compendious method may be employed.

Problem 6. (Converse of Prob. 4.)

The price, in shillings, of a specified number of articles, being given (the number being a multiple of 12*), to find the price of one article.*

Rule.—Regard the shillings in the price as so many pence, and divide this number of pence by the number of twelves in the specified number of articles: the quotient will be the price of a single article.

For to get the price of one, by the ordinary rule, we must divide the price of the entire number of articles by that number. Here the dividend would be shillings, and the divisor a multiple of 12. But since we may take a twelfth part of each, we may regard the shillings in the dividend as pence (a penny being $\frac{1}{12}$ of 1*s*.), and the divisor as $\frac{1}{12}$th of the divisor actually given: and hence the rule.

Examples.

1. If 48 pairs of scissors cost £1 4*s*., what will one pair cost?
   As there are 4 twelves in 48, therefore 24*d*. ÷ 4 = 6*d*., the *Ans*.
2. 72 yards of cloth cost £3 6*s*.; what is the cost per yard?
   72 = 12 × 6; therefore 66*d*. ÷ 6 = 11*d*., the *Ans*.
3. 48 articles for £1 16*s*.: required the cost of one?
   48 = 12 × 4: therefore 36*d*. ÷ 4 = 9*d*., the *Ans*.
4. 60 finger-plates for £7 10*s*.; what is the cost of one?
   60 = 12 × 5, therefore 150*d*. ÷ 5 = 30*d*. = 2*s*. 6*d*., the *Ans*.
5. If 84 articles cost £7 13*s* 6*d*., what is the cost of one?
   84 = 12 × 7; and £7 13*s*. 6*d*. = 153½*s*.; therefore,
   153½*d*. ÷ 7 = 21⅞*d*. + ⁴⁄₇*f*. = 1*s*. 10*d*., very nearly; the *Ans*.
6. If 48 articles cost £1 5*s*. 9*d*., what is the cost of one?
   48 = 12 × 4; and £1 5*s*. 9*d*. = 25¾*s*., and 25¾*d*. ÷ 4 = 6½*d*. nearly; the *Ans*.

Problem 7.

To calculate the price of any number of articles, when the price of one of them is less than a shilling.

Rule.—Regard the number of articles as so many pence, and multiply this number by the number of pence in the price of one article.

This rule, it will be seen, differs but little in its expression from that in common use; but instead of implicitly following the ordinary method, it will often be better, as here directed, to change the number of articles into so many pence, and the number of pence, in the price of one, into so many articles.

Examples.

1. Find the price of 48 lbs. at 9*d.* per lb.
   Here 4*s.* × 9 = £1 16*s.*, the *Ans.*
2. 84 lbs. at 7*d.* per lb? 7*s.* × 7 = £2 9*s.*, the *Ans.*
3. 132 lbs. at 11*d.* per lb.? 11*s.* × 11 = £6 1*s.*, the *Ans.*
4. 300 lbs. at 7*d.* per lb?
   300*d.* = £1 5*s.* and £1 5*s.* × 7 = £8 15*s.*, the *Ans.*
5. 65¼ oz. at 5*d.* per oz?
   65¼*d.* = 5*s.* 5¼*d.*, which × 5 = £1 7*s.* 2¼*d.*, the *Ans.*
6. 99⅛ yds. at 4*d.* per yd.?
   99⅛*d.* = 8*s.* 3⅛*d.*, 4 times which is £1 13*s.* 0½*d.*, the *Ans.*
7. 183⅝ yds. at 10*d.* per yd.?
   183⅝*d.* = 15*s.* 3⅝*d.*, and 10 times this is £7 13*s.* 0¼*d.*, the *Ans.*
8. 96 yds. at 10¾*d.* per yd.? 8*s.* × 10¾ = 86*s.* = £4 6*s.*, the *Ans.*
9. 75 lbs. at 9¾*d.* per lb? 75*d.* = 6*s.* 3*d.*; and 10 times this is—

	£3	2*s.*	6*d.*
¼ of 6*s.* 3*d.* =		1	6¾ (*Subtract.*)
Ans.	£3	0*s.*	11¼*d.*

10. What will 2½ dozen of edging come to, at 3¾*d.* per yd.?
    Take the price per yd. at 4*d.*; then—

30*d.* =	2*s.*	6*d.*
		4
	10	0 = price of 2½ doz at 4*d.*
¼ of 30*d.* =		7½ (*Subtract.*)
	9*s.*	4½*d.* = price at 3¾*d.*

We may proceed otherwise as follows: $3\frac{3}{4} = \frac{12}{4} + \frac{3}{4} = \frac{15}{4}$

Then, $\frac{15 \times 30}{4} = \frac{15 \times 15}{2} = \frac{225}{2} = 112\frac{1}{2}$ (pence) $= 9s.\ 4\frac{1}{2}d.$;

or, which is a little shorter, $\frac{15 \times 30}{4} = \frac{450}{4} = 112\frac{1}{2}$.

We shall now give a few miscellaneous examples for the practice of the reader in the foregoing rules; and one or two of the more complicated of these we shall solve at length; observing, however, that here, as well as in most of the worked examples that have preceded, an expert calculator may readily work mentally certain easy steps and operations, actually put down, and so reduce the space occupied, and, in a slight degree, the time expended.

Miscellaneous Examples.

1. Required the price of $47\frac{1}{4}$ dozen of Brussels lace, at $9s.\ 10\frac{9}{16}d.$ per yard.

   Here we have to perform this operation, namely, $118\frac{9}{16}s. \times 47\frac{1}{4}$; and we proceed thus:—

$$\tfrac{9}{16}s. = \frac{9 \times 12}{16}d. = \frac{27}{4}d. = 6\tfrac{3}{4}d. \quad \text{Also, } 47 = 9 \times 5 + 2.$$

Price of 1 doz. =	118s.	$6\frac{3}{4}d. \times 2.$	
(See Rule, p. 34.)		9	
	1067	$0\frac{3}{4}$	
		5	
	5335	$3\frac{3}{4}$	= price of 45 doz.
	237	$1\frac{1}{2}$	= ,, 2 doz.
	29	$7\frac{11}{16}$*	= ,, $\frac{1}{4}$ doz. (*Add.*)
	5602s.	$0\frac{15}{16}$	= price of $47\frac{1}{4}$ doz. = £280 2s. 1d.

2. $76\frac{3}{4}$ dozen of blonde lace, at $9s.\ 5\frac{7}{32}d.$ per yard?

   Here the work to be performed is $113\frac{7}{32}s. \times 76\frac{3}{4}$.

Now, $\frac{7}{32}s. = \frac{7 \times 12}{32}d. = \frac{21}{8}d. = 2\frac{5}{8}d.$ Also, $76\frac{3}{4} = 11 \times 7 - \frac{1}{4}$.

* $\frac{1}{4}$ of $30\frac{3}{4}d.$ is $7d. + \frac{2}{4}d. + \frac{3}{16}d. = 7d. + \frac{8}{16}d. + \frac{3}{16}d. = 7\frac{11}{16}d.$

Price of 1 doz. $= 113s.\ \ 2\frac{5}{8}d.$ $\quad[\frac{5}{8} \times 11 = \frac{55}{8} = 6\frac{7}{8}.]$

$$
\begin{array}{rll}
 & 113s. & 2\frac{5}{8}d. \\
 & & 11 \\
\hline
 & 1245 & 4\frac{7}{8} \\
 & & 7 \\
\hline
 & 8717 & 10\frac{1}{8} \quad = \text{price of } 77 \text{ doz.} \\
 & 28 & 3\frac{21}{32} \quad = \quad ,, \quad \frac{1}{4} \text{ doz. } (\textit{Subtract.}) \\
\hline
2,0) & 868,9s. & 6\frac{15}{32}d.^{*} = \text{price of } 76\frac{3}{4} \text{ doz.} \\
\hline
\text{or, } & £434\ 9s. & 6\frac{15}{32}d. \qquad [\frac{15}{32}d. = \frac{1}{2}d. \text{ very nearly.}]
\end{array}
$$

*** The statements within the brackets and in the foot-notes are needful hints only to those whose acquaintance with fractions is but slight.

3. $23\frac{1}{2}$ dozen of French lace, at $2s.\ 11\frac{5}{16}d.$ per yard?

$\frac{5}{16}s. = \frac{5 \times 12}{16}d. = 3\frac{3}{4}d.$ And $35s.\ 3\frac{3}{4}d. \times (24 - \frac{1}{2})$ is worked thus:

$$
\begin{array}{ll}
35s. & 3\frac{3}{4}d. = \text{price of 1 doz.} \\
 & 24 \quad \text{(See the work within brackets below.)} \\
\hline
847 & 6 \quad = \text{price of 24 doz.} \\
17 & 7\frac{7}{8} \quad = \quad ,, \quad \frac{1}{2} \text{ doz. } (\textit{Subtract}) \\
\hline
829s. & 10\frac{1}{8}d. = \quad ,, \quad 23\frac{1}{2} \text{ doz.}
\end{array}
$$

$= £41\ 9s.\ 10\frac{1}{8}d.$, the answer. The $\frac{1}{8}d.$ would, of course, be disregarded.

$[\frac{3}{4}d. \times 24 = 1s.\ 6d.$; and $3d. \times 24 = 6s.]$

$[35 \times 24 = 70 \times 12.]$

$[\frac{1}{2}$ of $3\frac{3}{4}d. = 1\frac{1}{2}d. + \frac{3}{8}d. = 1\frac{4}{8}d. + \frac{3}{8}d. = 1\frac{7}{8}d.]$

4. Required the cost of 150 silk mantles, at £1 3*s.* 9*d.* each?
   Ans. £178 2*s.* 6*d.*
5. If 11 dozen of wine glasses cost £4 19*s.*, what is the cost of one?
   Ans. 9*d.*
6. What is the value of $19\frac{1}{2}$ reams of paper, at $7s.\ 9\frac{3}{4}d.$ per ream?
   Ans. $£7\ 12s.\ 4\frac{1}{8}d.$
7. What is the value of $13\frac{3}{4}$ gallons of rum, at $15s.\ 9\frac{1}{2}d.$ per gallon?
   Ans. $£10\ 17s.\ 1\frac{5}{8}d.$
8. Required the price of 85 lbs. of beef, at $9\frac{3}{4}d.$ per lb.?
   Ans. $£3\ 9s.\ 0\frac{3}{4}d.$
9. If £1 4*s.* $9\frac{1}{2}d.$ be paid for $42\frac{1}{2}$ lbs. of beef, what is the cost per lb.?
   Ans. 7*d.*
10. If £3 3*s.* $7\frac{3}{4}d.$ be paid for $23\frac{1}{2}$ yards of silk, what is the cost per yard?
    Ans. $2s.\ 8\frac{1}{2}d.$
11. What is the price of $68\frac{1}{2}$ lbs. of worsted, at $17\frac{1}{2}d.$ per lb.?
    Ans. $£4\ 19s.\ 10\frac{3}{4}d.$

---

* $\frac{1}{8} - \frac{21}{32} = \frac{4}{32} - \frac{21}{32}$; and, borrowing $\frac{32}{32}$, or 1, $\frac{36}{32} - \frac{21}{32} = \frac{15}{32}$; and the 1 *borrowed must be carried to the 3.*

12. What is the price of two dozen yards of Nottingham lace, at $2s.\ 8\frac{1}{8}d.$ per yard? *Ans.* £3 $4s.\ 3d.$
13. What is the value of 12 oz. of silver, at $4s.\ 7\frac{3}{16}d.$ per oz.? *Ans.* £2 $15s.\ 2\frac{1}{4}d.$
14. What is the price of $16\frac{1}{4}$ doz. of lace, at $5s.\ 7\frac{3}{8}d.$ per yard? *Ans.* £54 $9s.\ 10\frac{1}{8}d.$
15. Calculate the value of $27\frac{3}{4}$ doz. of lace, at $3s.\ 2\frac{5}{32}d.$ per yard. *Ans.* £53 $2s.\ 3\frac{21}{32}d.$
16. Find the value of $127\frac{1}{2}$ doz. of lace, at $2s.\ 1\frac{15}{16}d.$ per yard. *Ans.* £165 $7s.\ 0\frac{3}{8}d.$
17. If a dozen yards cost £5 $10s.\ 4\frac{1}{2}d.$, what is the cost per yard? *Ans.* $9s.\ 2\frac{3}{8}d.$ [$\frac{3}{8}d. = \frac{1}{4}d. + \frac{1}{2}f.$].
18. If a dozen articles cost £2 $7s.\ 8d.$, what is the cost of one? *Ans.* $3s.\ 11\frac{2}{3}d.$ [$\frac{2}{3}d. = \frac{1}{2}d. + \frac{2}{3}f.$].

In the foregoing problems and examples, the articles concerned are reckoned in dozens; and, whenever the number of them is not an exact number of dozens, it has been shown how the excess or defect is to be calculated and allowed for. But in certain commercial transactions, the commodities are sold by the gross (twelve dozen), or by the score (twenty); and in others, by the 100 (five score), or by the 120 (six score). The following problems relate to purchases in which the articles are enumerated in one or other of these ways.

Problem 8.

To find the price of a gross, the price of one of the articles being given in pence.

Rule.—Regard the pence in the price of one article as so many shillings. Multiply these shillings by twelve, and the product will be the price of a gross.

For, by taking the pence as so many shillings, the price is, in effect, multiplied by 12; so that, by again multiplying by 12, the product will be 144 times the price of one article; that is, it will be the price of twelve dozen, or a gross.

Examples.

1. A gross, at $8\frac{1}{4}d.$ each article, is $8s.\ 3d. \times 12 = 99s. =$ £4 $19s.$
2. A gross, at $9\frac{1}{2}d.$ each, is $9s.\ 6d. \times 12 = 114s. =$ £5 $14s.$
3. A gross, at $11\frac{3}{4}d.$ each, is $11s.\ 9d. \times 12 = 141s. =$ £7 $1s.$
4. A gross, at $13\frac{1}{2}d.$ each, is $13s.\ 6d. \times 12 = 162s. =$ £8 $2s.$
5. A gross, at $19\frac{1}{4}d.$ each, is $19s.\ 3d. \times 12 = 231s. =$ £11 $11s.$
6. A gross, at $23\frac{3}{4}d.$ each, is £1 $3s.\ 9d. \times 12 =$ £14 $5s.$

We need scarcely observe, in reference to the second method, that the multiplication of the pence by 12 need never be actually performed; since we see, at once, that the product is the same number of shillings as the number of pence.

10. What is the price of 301 yards of silk, at 17*s.* 9½*d.* per yard?

17*s.* 9½*d.* = 213½*d.*				
£213½ =	£213	10	0*d.*	= price of 240.
¼ of £213 10*s.* =	53	7	6*d.*	= price of 60.
		17	9½*d.*	= price of 1.
	£267	15*s.*	3½*d.*	= price of 301.

Otherwise:

£17¾ + 10*d.* =	£17	15*s.*	10*d.*,	one score.
			10	
Price of 200 =	177	18	4	
Price of 100 =	88	19	2	
Price of 1 =		17	9½	
Price of 301 =	£267	15*s.*	3½*d.*	

11. 240 yards of cloth, at 16*s.* 7¾*d.* per yard? *Ans.* £199 15*s.*
12. 260 articles, at 23*s.* 7¼*d.* each? *Ans.* £306 17*s.* 1*d.*
13. 280 yards, at 27*s.* 9¾*d.* per yard? *Ans.* £389 7*s.* 6*d.*
14. 400 yards of silk velvet, at 21*s.* 7¾*d.* per yard? *Ans.* £432 18*s.* 4*d.*
15. 720 lbs., at 7¼*d.* per lb.? *Ans.* £21 15*s.*
16. 960 stone, at 10½*d.* per stone? *Ans.* £42.
17. 1440 lbs., at 3*s.* 9*d.* per lb.? *Ans.* £270.
18. 1680 lbs., at 5*s.* 5½*d.* per lb.? *Ans.* £458 10*s.*
19. 967 lbs., at 4½*d.* per lb.? *Ans.* £18 2*s.* 7½*d.*
20. 1199¾ lbs., at 5*s.* 5½*d.* per lb. *Ans.* £327 8*s.* 7⅝*d.*

PROBLEM 10.

To find the price of a score, when the price of one article is given in pence.

This problem has been, in substance, anticipated in the observations within the brackets at page 62, but as no Rule is formally given in that place, we supply the Rule here.

RULE I.—Regard the pence as so many pounds, and divide that sum by 12: the result will be the price of a *score.*

EXAMPLES.

1. A score, at $5\frac{1}{4}d.$ each, is £5 $5s. \div 12 = 105s. \div 12 = 8s.\ 9d.$
2. A score, at $13\frac{1}{2}d.$ each, is £13 $10s. \div 12 = 270s. \div 12 = 22s.\ 6d.$
3. Three score, at $7\frac{3}{4}d.$, is £7 $15s. \times 3 \div 12 =$ £23 $5s. \div 12 =$ £24 $\div$ 12, *less* $1s.\ 3d. =$ £1 $18s.\ 9d.$ [$15s. \div 12 = 1s.\ 3d.$]
   Or: £7 $15s. \times \frac{3}{12} =$ £7 $15s. \div 4 =$ £1 $18s.\ 9d.$

As there is little or no advantage in this, over the common method, we shall not exemplify it further. Indeed, the common method, when slightly modified as follows, seems to be the preferable of the two.

RULE II.—Multiply the pence by 20, and add $5d.$ for every farthing: the result will be the price in pence: thus, taking the preceding examples, in order.

1. $5d. \times 20 + 5d. = 105d. = 8s.\ 9d.$
2. $13d. \times 20 + 10d. = 270d. = 22s.\ 6d.$
3. $7d. \times 20 + 15d. = 155d. = 12s.\ 11d.$, which multiplied by 3 gives £1 $18s.\ 9d.$

NOTE.—When the price of the single article consists of shillings and pence, the pence may be calculated for by one or other of these rules; and then the shillings, taken as pounds, be prefixed: for example, if the price of one article were $3s.\ 5\frac{1}{4}d.$, the price of a score of the articles would be £3 $8s.\ 9d.$ (See Ex. 1 above.) Again: in Ex. 2, the price of 1 is $1s.\ 1\frac{1}{2}d.$: at $1\frac{1}{2}d.$ only, the price of 20 is seen at once to be $2s.\ 6d.$: hence, at $1s.\ 1\frac{1}{2}d.$, the price of a score is £1 $2s.\ 6d.$

PROBLEM 11.

To find the price of 100 *articles, when the price of one of them is given.*

RULE.—1. Multiply the shillings (if there be any) in the given price, by 5; the result will be pounds.

2. For every farthing in the pence, take twice as many shillings, and once as many pence; their sum added to the pounds will be the price required.

The reason is this:—Twenty times any number of shillings are so many pounds; and therefore 100 times that number of shillings must be 5 times that number in pounds. Again: taking the farthings for so many pence, is the same as multiplying the farthings by 4; and taking each of these farthings for $2s.$, or 96 farthings, is the same as multiplying them by 96; so that, working in this way, we multiply the farthings by $4 + 96$, that is, by 100, as we are required to do.

EXAMPLES.

1. 100 articles at 2*s*. $3\frac{1}{4}$*d*. each?
In the pence here, there are 13 farthings: hence, by the rule, we have,

	£	*s*.	*d*.
For the 2*s*. . . .	10	0	0
For the 13 *far*. . {	1	6	0
		1	1
Ans.	£11	7*s*.	1*d*.

2. 100 at 17*s*. $10\frac{1}{4}$*d*. each?

	£	*s*.	*d*.
For the 17*s*. . . .	85	0	0
For the 41 *far*. . {	4	2	0
		3	5
Ans.	£89	5*s*.	5*d*.

3. 100 yards, at $5\frac{3}{4}$*d*. per yd?

	£	*s*.	*d*.
For 23 *far*. . . {	2	6	0
		1	11
Ans.	£2	7*s*.	11*d*.

4. 100 articles at 7*s*. $6\frac{3}{4}$*d*. each.

	£	*s*.	*d*.
For the 7*s*. . . .	35	0	0
For the 27 *far*. . {	2	14	0
		2	3
Ans.	£37	16*s*.	3*d*.

5. 100 at $14\frac{1}{2}$*d*. each?
£5 + 20*s*. + 10*d*. =
£6 0*s*. 10*d*. *Ans*.

6. 100 copy-books, at $4\frac{1}{2}$*d*. each? *Ans*. £1 17*s*. 6*d*.
7. 100 quarts of vinegar at 1*s*. $3\frac{3}{4}$*d*. per quart? *Ans*. £6 11*s*. 3*d*.
8. 100 articles at £2 7*s*. 6*d*. each? *Ans*. £239 10*s*.
9. 100 at 11*s*. 4*d*. each? *Ans*. £283 6*s*. 8*d*.
10. 100 at 2*s*. $4\frac{1}{4}$*d*. each? *Ans*. £11 15*s*. 5*d*.

If we have to perform the reverse operation, namely, from having the price of 100 articles, to find the price of one of them, the common method of working is as convenient and as expeditious as can be desired; that is, taking the converse of Ex. 4, the method exhibited in the margin; which gives for result, 7*s*. $6\frac{3}{4}$*d*. (See Problem 13.)

£	*s*.	*d*.
37	16	3
20		
7,56		
12		
6,75		
4		
3,00		

PROBLEM 12.

To find the price of 120 *articles, the price of one of them being given.*

RULE.—Half the number of pence in the price of one, taken as so many pounds, will be the price in £ of 120. For, as already shown (page 62), the whole number of *pence*, taken as so many pounds, is the price of 240.

Examples.

1. What is the price of six score lambs, at 12*s*. 6*d*. each?
   £150 ÷ 2 = £75, *Ans*.
2. 120 articles, at 3*s*. 8¾*d*. each? £44 15*s*. ÷ 2 = £22 7*s*. 6*d*., *Ans*.
3. 120, at 11*s*. 5¼*d*. each? £137 5*s*. ÷ 2 = £68 12*s*. 6*d*., *Ans*.
4. 120, at £1 2*s*. 7½*d*.? £120 + ½ (£31 10*s*.) = £135 15*s*., *Ans*.
5. What is the price of six score flower-pots, at 4*d*. each? *Ans*. £2.
6. What is the price of six score brass finger-plates, at 2*s*. 6*d*. each? *Ans*. £15.
7. 120 pairs of gloves, at 2*s*. 3½*d*. per pair? (See Ex. 5, p. 55.) *Ans*. £13 15*s*.
8. 120 gallons of rum, at 13*s*. 10*d*. per gallon? (See Ex. 6, p. 55.) *Ans*. £83.

For the reverse problem, that is, to find the price of one from the price of 120 being given, we may proceed thus: take 5 times the given price, and, for the moment, regard the product as the price of 100 articles, and compute for one, as shown above, and then divide the result by 6. Thus, taking the converse of Ex. 2, the work is that in the margin. [See, however, Prob. 13.]

£	s.	d.
22	7	6
		5
111	17	6
20		
22,37		
12		
4,50		
4		
2,00		
6) 22*s*. 4½*d*.		
3*s*. 8¾*d*.		

The reason of this is, that ⅚ths of 120 is 100, so that any sum divided by 120 is the same as ⅚ths of that sum divided by 100; that is, the quotient in the former case is ⅚ths of the quotient in the latter case.

It is as well to mention here that when, in either of the two reverse operations in the margin above, the figure cut off for farthings leaves a significant number on the right, instead of ciphers as in the foregoing results, that number in the first case expresses so many hundredths of a farthing, and in the second case so many 120ths: we thus get the price of the single article to the minutest accuracy, and can then increase or diminish the exact price, thus determined, by a farthing, according as this overplus fraction is greater or less than half a farthing; that is, according as the number cut off to the right is greater or less than 50, in the first case (p. 66,) or greater or less than 60, in the second case. But whenever such extreme accuracy is considered to be unnecessary, and the price of the 100 or of the 120 is in pounds, the price of one, within a farthing of error, may be quickly found by the following rule.

Problem 13.

The price of 100 *or of* 120 *being given in pounds, to find the price of one either exactly, or to the nearest farthing.*

Rule 1.—For 100. Take the price in pounds as so many shillings, and divide by 5. [For this is the same as dividing by 20 and 5.]

2. For 120. Take the price in pounds as so many shillings, and divide by 6. [For this is the same as dividing by 20 and 6.]

Examples.

1. If 100 articles cost £28 10*s.*, what is the cost of one?
   28*s.* 6*d.* ÷ 5 = 5*s.* 8$\frac{3}{5}$*d.* exactly.

2. If 100 cost £37 16*s.* 3*d.*, what is the cost of one? [This is worked to strict accuracy in the margin at page 66.]

That the given sum may be expressed in pounds (nearly), without an inconvenient fraction, we may regard it as £37 15*s.* = £37$\frac{3}{4}$; then 37*s.* 9*d.* ÷ 5 = 7*s.* 6$\frac{3}{5}$*d.*, exact to the nearest farthing; for $\frac{3}{4}$ differs from $\frac{3}{5}$ only by $\frac{3}{4} - \frac{3}{5} = \frac{15}{20} - \frac{12}{20} = \frac{3}{20}$ths of a penny: we therefore replace $\frac{3}{5}$*d.* by $\frac{3}{4}$*d.*

3. If 120 cost £22 7*s.* 6*d.*, what will be the cost of one? (See Example 2, p. 67.)

We see by the Table (p. 50), that 6*s.* 8*d.* = £$\frac{1}{3}$ and that 10*d.* = £$\frac{1}{24}$. Also that $\frac{1}{3}$*s.* + $\frac{1}{24}$*s.* = 4$\frac{1}{2}$*d.*: hence 22*s.* 4$\frac{1}{2}$*d.* ÷ 6 = 3*s.* 8$\frac{3}{4}$*d.*, the *Ans.*

4. If 120 cost £68 12*s.* 6*d.*, required the cost of one? (See Ex. 3, p. 67.)

Taking the sum to be £68$\frac{1}{2}$, we have 68*s.* 6*d.* ÷ 6 = 11*s.* 5*d.*, a farthing short; but we have taken the given cost £$\frac{1}{8}$ too little; and $\frac{1}{8}$*s.* = 1$\frac{1}{2}$*d.*, $\frac{1}{6}$th of which gives the wanting farthing.

Note.—It is plain that in order to convert the shillings and pence in the given sum into a convenient fraction of a pound, we need never increase or diminish these shillings and pence by more than (nor even by so much as) 2*s.* 6*d.* ($\frac{1}{8}$th of a pound), so that we shall never have to allow more than $\frac{1}{4}$*d.* for the excess or defect whether there be 100 articles or 120; for in the former case $\frac{1}{5}$th of 1$\frac{1}{2}$*d.* differs from a farthing by only $\frac{1}{5}$ of a farthing, and in the latter case $\frac{1}{6}$th of 1$\frac{1}{2}$*d.* is a farthing exactly. In Example 3, above, since 7*s.* 6*d.* is three half-crowns, it is £$\frac{3}{8}$, so that the given sum is exactly £22$\frac{3}{8}$, and therefore the exact price of the single article is, as stated, 22*s.* 4$\frac{1}{2}$*d.* ÷ 6 = 3*s.* 8$\frac{3}{4}$*d.* In like manner, in Example 4, the given sum is exactly £68$\frac{5}{8}$; and 68*s.* 7$\frac{1}{2}$*d.* ÷ 6 = 11*s.* 5$\frac{1}{4}$*d.*, the price exactly. We have worked this example, and Example 2, as above, in order that the reader may clearly see how inconsiderable is the effect, upon the result, by taking the given sum either in excess or in defect to the extent of even half-a-crown; for which extreme sum only a farthing is to be allowed as *correction* of the error.

It will be of service to the retailer to recollect that if he buy articles by the long hundred (six score) he pays for each article as many farthings as he lays out half crowns; and that if he buy by the common hundred (five score) he pays at the rate of $1\frac{1}{2}d.$ for five articles, for every half-crown he lays out. For 120 farthings $= 30d. = 2s.\ 6d.$; and $1\frac{1}{2}d. \times 20 = 30d. = 2s.\ 6d.$; and in 100 there are 20 fives. [Ex. 7, below: £13 15*s.* = 110 half-crowns; and 110 far. $= 2s.\ 3\frac{1}{2}d.$]

5. If 100 cost £56 13*s.* 4*d.*, what is the cost of one? *Ans.* 11*s.* 4*d.*
6. If 100 cost £6 11*s.* 3*d.*, what does one cost? *Ans.* 1*s.* $3\frac{3}{4}d.$
7. If 120 cost £13 15*s.*, what is the cost of one? *Ans.* 2*s.* $3\frac{1}{2}d.$
8. If 120 cost £135 10*s.*, what is the cost of one? *Ans.* £1 2*s.* $7\frac{1}{2}d.$

Problem 14.

To find the price of 1000, *the price of one being given in pence.*

Rule.—Regard the price in pence as pounds, and multiply these pounds by $4\frac{1}{6}$: the product will be the price of 1000.

For the pence, taken as pounds, gives the price of 240 (page 62). Four times 240 is 960; and $\frac{1}{6}$th of 240, that is, 40 added to this, gives 1000. [It may be useful to recollect that 1000, at 1*d.*, amount to £4 3*s.* 4*d.*, and therefore, at 2*d.*, to £8 6*s.* 8*d.*; and so on.]

Examples.

1. At $1\frac{3}{4}d.$ per yard, what will 1000 yards cost?
   £$1\frac{3}{4}$ is £1 15*s.*, which $\times\ 4\frac{1}{6} =$ £7 5*s.* 10*d.*, the *Ans.*
2. At 7*s.* 9*d.* per yard, 1000 yards is
   £93 $\times\ 4\frac{1}{6} =$ £372 + £15 10*s.* = £387 10*s.*

Note.—In this example we have written down, first the product by 4, and have then connected with it the product by $\frac{1}{6}$, making two separate amounts, which are then added into one sum. But in the first example we have written this final sum at once, since we could readily determine it without thus calculating the two component parts of it separately. We took, for the multiplier 4 only, not the £1 15*s.*, but £$1\frac{3}{4}$, and wrote £7 for the product; we then divided the equivalent of this £$1\frac{3}{4}$, namely, 35*s.*, by 6, and annexed the resulting 5*s.* 10*d.* to the previously written £7. And advantage should always be thus taken of every little peculiarity favourable to the abbreviation of the figure-work.

3. At 2*s.* $7\frac{1}{2}d.$ per yard, what will 1000 yards cost?
   £$31\frac{1}{2}$ = £31 10*s.*, which $\times\ 4\frac{1}{6}$ is £126 + £5 5*s.* = £131 5*s.*, *Ans.*
   Here $31\frac{1}{2} \times 4 = 126$.
4. At 14*s.* 7*d.* per gallon, what will 1000 gallons cost?
   £175 $\times\ 4\frac{1}{6} =$ £700 + £29 3*s.* 4*d.* = £729 3*s.* 4*d.*, *Ans.*
5. 1000 yards, at $3\frac{1}{2}d.$? *Ans.* £14 11*s.* 8*d.*

6. 1000 gallons, at 3*s*. $10\frac{1}{2}$*d*.? *Ans*. £193 15*s*.
7. 1000 at 1*s*. $3\frac{3}{4}$*d*. each? *Ans*. £65 15*s*.
8. 1000 at $5\frac{1}{4}$*d*. each? *Ans*. £21 17*s*. 6*d*.
9. 1000 at 3*s*. $7\frac{3}{4}$*d*. each? *Ans*. £182 5*s*. 10*d*.
10. 1000 at 1*s*. $11\frac{3}{4}$*d*. each? *Ans*. £98 19*s*. 2*d*.

NOTE.—We might compute this example for 2*s*.; and knowing that 960 farthings make £1, and that 40 make 10*d*., we should have to subtract £1 0*s*. 10*d*. from the result.

PROBLEM 15 (CONVERSE OF PROB. 14).

The price of 1000 *being given in pounds, to find the price of one.*

RULE.—Regard the price in pounds as so many pence, and divide these pence by $4\frac{1}{6}$. Or, which is the same thing, multiply the pounds, taken as pence, by 6, and divide the result by 25. For $4\frac{1}{6} = \frac{25}{6}$; and to divide by a fraction is to turn the fraction upside down (or to reverse its terms) and multiply; so that to divide by $4\frac{1}{6}$, or by $\frac{25}{6}$, is to multiply by $\frac{6}{25}$.

EXAMPLES.

1. If 1000 cost £25, what is the cost of one?
$25d. \times \frac{6}{25} = 6d.$, the *Ans*.

2. If 1000 cost £387 10*s*., what is the cost of one?
$387\frac{1}{2}d. \times 6 = 2325d.$
5) 2325*d*.
5) 465
93*d*. = 7*s*. 9*d*., *Ans*.

3. If 1000 cost £131 5*s*., what is the cost of one?
$131\frac{1}{4}d. \times 6 = 787\frac{1}{2}d.$
5) 1575 halfpence.
5) 315
$63 = 31\frac{1}{2}d. = 2s. 7\frac{1}{2}d.$, *Ans*.

4. If 1000 cost £729 3*s*. 4*d*., what will one cost?
$729\frac{1}{6}d. \times 6 = 4375d.$
5) 4375
5) 875
Ans. 175*d*. = 14*s*. 7*d*.

5. If 1000 cost £182 5*s*. 10*d*., what will one cost?
Omit the 10*d*. for the present; then we have—
$182\frac{1}{4}d. \times 6 = 1093\frac{1}{2}d.$
5) 2187 halfpence.
5) 437,2 rem.
87,2 rem.
that is, $43\frac{1}{2}d. + \frac{12}{25}$ of 1 halfpenny or $\frac{24}{25}$ far.

Now 1000th part of the omitted 10*d.* is $\frac{1}{100}d. = \frac{4}{100}$ far., and $\frac{24}{25}$ far. $= \frac{96}{100}$ far.

The sum of these two is $\frac{100}{100}$ far. $= 1$ farthing: hence the price required is $43\frac{3}{4}d. = 3s.\ 7\frac{3}{4}d.$ (See Ex. 9, Prob. 15.)

It is obvious that this supplementary work for the accurate estimate of the influence of the omitted 10*d.*, and of the fraction $\frac{24}{25}f.$, upon the cost price of one of the articles, might have been dispensed with. The antecedent result, namely, $43\frac{1}{2}d. + \frac{24}{25}f.$, is sufficiently conclusive that the sought price, per article, is either $43\frac{3}{4}d.$ exactly, or else this sum to the nearest farthing. No account at all need ever be taken of the odd pence in the wholesale price; nor even of the shillings, below 10*s.*, if the object be only to find the cost price, per article, to the nearest farthing. For we have just seen that the influence of 10*d.* on this cost price is but $\frac{4}{100}f.$; so that the influence of 12 times 10*d.*, or 10*s.*, would be only $\frac{48}{100}f.$, which is less than half a farthing. Hence when the prime cost of 1000 articles is so many pounds and shillings, if the shillings do not exceed 10*s.*, we may disregard them, and may calculate the prime cost of a single article to the nearest farthing, by taking account of the pounds only in the cost of the 1000; and if the shillings exceed 10*s.*, then we have only to add 1 to the number of pounds, and to calculate as above.

The principal object in finding the prime cost of each one of a large number of articles, purchased wholesale, is to guide the retail dealer as to what he ought to charge per article, selling them singly, in order to realize what he considers a reasonable profit upon his outlay: the way to find the selling price per article, with a view to such profit, will be shown in the next problem.

Let us now return to Ex. 5, disregarding not only the 10*d.*, but also the 5*s.*: the work will then be as follows:—

$182d. \times 6 = 1092d.$; this $\div 5 = 218d.$, with 2 for remainder; and $218d. \div 5 = 43d.$, with 3 for remainder: hence the complete remainder, arising from the division by 25, is 17; so that the complete result is, $43\frac{17}{25}d. = 43\frac{3}{4}d.$, to the nearest farthing; that is, it is impossible that this result can be so much as a farthing in error; for $\frac{17}{25}d.$ is itself more than $\frac{1}{2}d.$, it is in fact $\frac{1}{2}d.$ and $\frac{4\frac{1}{2}}{25}d.$, that is, $\frac{18}{25}f.$ besides; and then there is the omitted 5*s.* 10*d.* unaccounted for.

The reader is expected to work the subjoined examples in accordance with what has now been said, always assuming pounds only, for the cost price, without shillings or pence.

Thus, in Examples 6, 8, and 10, the cost price of the 1000 is to be regarded as £165, £388, and £18, respec-

tively; and in the other examples the odd shillings and pence are to be omitted.

6. If 1000 articles cost £164 18*s.*, what is the cost of one, to the nearest farthing? *Ans.* 3*s.* $3\frac{1}{2}$*d.*
7. If 1000 cost £325 9*s.*, what is the cost of one, to the nearest farthing? *Ans.* 7*s.* 6*d.*
8. If 1000 cost £387 16*s.*, what is the cost of one, to the nearest farthing? *Ans.* 7*s.* 9*d.*
9. If 1000 cost £729 6*s.* 6*d.*, what is the cost of one, to the nearest farthing? *Ans.* 14*s.* 7*d.*
10. If 1000 cost £17 18*s.* 9*d.*, what is the cost of one, to the nearest farthing? *Ans.* $4\frac{1}{4}$*d.*

Problem 16.

From knowing the cost-price of any number of articles, to find the selling-price, per article, so that a certain amount of profit may be realized.

A trader accustomed to purchase the articles in which he deals by the 100, or the 120, or the 1000, &c., can form a pretty close guess, from practice, of the prime cost of one of the articles from his outlay for the entire number; but even should his approximation be comparatively wide of the truth, it will still suffice for his purpose. In proof of this, let us return to Example 5, of last problem, where £182 5*s.* 10*d.* is supposed to be paid for 1000 articles; and let us assume that the purchaser estimates the prime cost, per article, at about 3*s.* 6*d.*: his business will then be to find, by problem 14, the price of 1000 at this rate; that is, he will have to calculate thus:

$$£42 \times 4\tfrac{1}{6} = 168 + £7 = £175.$$

He would in this way find that 3*s.* 6*d.* per article is less than the cost price; that, in fact, he has paid for the 1000, more by £7 5*s.* 10*d.* than he would have paid if the price had been only 3*s.* 6*d.* per article: his aim would therefore now be gradually to increase this 3*s.* 6*d.* till it amounts to a retail price sufficiently high to cover the outlay and give a reasonable profit besides. Now as 1000 farthings is £1 0*s.* 10*d.*, it is only necessary to see how many times this sum, when added to £175, will produce an amount above *the outlay,* £182 5*s.* 10*d.*, sufficient for the profit proposed.

It is plain, at a glance, that 6 times £1 0*s.* 10*d.* will not be sufficient; taking 7 times, we have, at the rate of 3*s.* 7$\frac{3}{4}$*d.* each, the price of 1000 = £175 + £7 5*s.* 10*d.* = £182 5*s.* 10*d.* It thus so happens, in this particular case, that the cost price, per article, is an exact sum, without fractions of a farthing, namely, 3*s.* 7$\frac{3}{4}$*d.*; and we may here notice, that whenever such is the case, we may, in this manner, always arrive at the cost price.

Adding now farthing after farthing to the 3*s.* 7$\frac{3}{4}$*d.*, we see that the profit of the 1000, at 3*s.* 8*d.* each, is £1 0*s.* 10*d.*; at 3*s.* 8$\frac{1}{4}$*d.*, it is £2 1*s.* 8*d.*; at £3 9*d.* (that is, adding five farthings), it is £5 4*s.* 2*d.*, and so on; £1 0*s.* 10*d.* being the additional profit for every additional farthing. If it had so happened that 3*s.* 7$\frac{3}{4}$*d.* was not the exact price, as would have been the case if the outlay had been (say) £182 only, then, with the profits here determined, there would have been, in each case, 5*s.* 10*d.* profit besides.

It is obvious that whatever be the number of articles purchased wholesale for a given sum, the profit to the retailer upon his outlay, by selling the articles singly, at any assigned price, may in this way always be easily determined. If the estimated cost price, per article, be but a few pence, we may, as above, readily find the profit to be realized by increasing this estimated cost price by any number of farthings; or if the cost price be considerable, by any number of pence, or even shillings; the general rule being this:—

Rule.—Having roughly estimated the prime cost of the single article, find by the suitable rule (among the rules already given), the exact cost of the entire number, at this estimated price.

Then, since every farthing, halfpenny, penny, &c., added to the price of one article, adds as many farthings, halfpence, pence, &c., as there are articles, to the price of the whole, we can easily find what must be the increase, in the above estimated cost price of one, in order that the price of the whole, at that rate per article, may exceed the actual outlay by the desired amount of profit; as in the following examples.

Examples.

1. If a dozen articles cost £2 7*s.* 8*d.*, what is the lowest retail price,

per article, that will produce a profit upon the outlay of not less than 10s. 6d.?

Take the estimated cost price, per article, at 4s.; then—

at 4s., the price of 12 would be			£2 8s.
,, 4s. 6d.	,,	,,	£2 14s.
,, 4s. 9d.	,,	,,	£2 17s.
,, 4s. 10½d.	,,	,,	£2 18s. 6d.

and £2 18s. 6d. less £2 7s. 8d. = 10s. 10d.; so that, to gain this profit, the articles must be sold at 4s. 10½d. each. Of course, from 4s. to 4s. 10½d. might be reached at once by adding 10½s. to the £2 8s.

2. If 150 articles cost £178 2s. 6d., what is the lowest price at which each must be sold, so that the retailer may realize at least £12 profit upon his outlay?

Take the estimated cost price, per article, at £1; then, bearing in mind that 150s. = £7 10s., we see that

at £1, the cost would be			£150			
,, £1 5s.	,,	,,	£187	10s.		
,, £1 5s. 6d.	,,	,,	£191	5s.		
			£178	2s.	6d.	(Subtract.)
			£13	2s.	6d.	Profit at £1 5s. 6d.

If we take 150d. = 12s. 6d. from this there will remain £12 10s.; hence for this profit, the retail price must be £1 5s. 5d.; or, deducting 6s. 3d. (the half of 12s. 6d.) from the £12 10s., there will remain £12 3s. 9d., the profit when the retail price is £1 5s. 4½d.; and by the deduction of another farthing from the retail price, the profit is reduced to £12 0s. 7½d. And in this way may the increase or decrease of profit, consequent upon any increase or decrease in the retail price per article, be always readily found.

3. If 19¼ reams of paper be purchased for £7 12s. 4d., at what price, per ream, must it be sold, in order that about 30s. profit upon the outlay may be realized?

Take the estimated cost price at 8s. per ream: then,

				£	s.	d.
at 8s., the cost price of 19½ reams would be				7	16	0
,, 9s.	,,	,,	,,	8	15	6
,, 9s. 6d.	,,	,,	,,	9	5	3
			Subtract	7	12	4
			Profit, at 9s. 6d. per ream,	£1	12s.	11d.

Taking 19½d. from this, the profit, at 9s. 5d. per ream, will be £1 11s. 3½d., which is probably near enough; but if the retail price be further reduced ½d., the profit would be 9¾d. less, namely, £1 10s. 5¾d. And at 9s. 4½d., it would be £1 10s. 1d. Of course there is another *way of working* questions of this kind: we may add the proposed profit

to the outlay, and then, either by Prob. 6, or by common division of this sum by the number of articles, find the selling price of one of them.

But by proceeding otherwise than as above, fractions of a farthing—things having no representation in current coin—are not unlikely to occur in the result; and the neglect of the fraction, though in an individual case of but very trifling consequence, yet, in a large number of sales, by the single article, such neglect may cause a deficiency of considerable amount.

For variety, we shall here work the foregoing example by ordinary division.

£	*s.*	*d.*	
7	12	4	Cost price of $19\frac{1}{2}$ reams.
1	10	0	Profit to be made.
9	2	4	Selling price of the $19\frac{1}{2}$ reams.

$$\begin{array}{r} 9 \\ 20 \\ \hline 182 \\ 12 \\ \hline 2188 \\ 2 \\ \hline \end{array}$$

$$39)\,4376\,(112\tfrac{8}{39}d.$$

[Both divisor and dividend are doubled, in order to do away with the fraction $\frac{1}{2}$ in the former.]

$$\begin{array}{r} 39 \\ \hline 47 \\ 39 \\ \hline 86 \\ 78 \\ \hline 8 \\ \hline \end{array}$$

Therefore reducing the fraction of a penny to that of a farthing, we have, for the selling price, 9*s.* 4*d.* + $\frac{32}{39}$*f.*, or 9*s.* $4\frac{1}{4}$*d.* all but $\frac{7}{39}$*f.*

4. If 83 yards of lace cost £53 19*s.*, what must it be sold at per yard for the profit on the outlay to be £5 9*s.*? *Ans.* 14*s.* $3\frac{3}{4}$*d.*
5. If 107 lbs. of tea cost £21 3*s.*, at what rate per lb. must it be sold, so that £4 14*s.* profit may be gained? *Ans.* 4*s.* 10*d.*
6. If 23 tons of potatoes be purchased for £56 12*s.*, at what price, per ton, must they be sold, to produce a profit of £11 4*s.* on the outlay? *Ans.* £2 19*s.* all but $\frac{1}{2}$*d.*

NOTE.—In here concluding the foregoing series of problems, it may be well to state that although the rules proposed for the working of them are all specially adapted to the ready computation of the several kinds of arithmetical examples considered; yet that a case or two, properly coming under one or other of these problems, may occasionally occur in which the ordinary process of common arithmetic may answer quite as well as the generally shorter method here prescribed. It must always be left for the judgment and penetration of the computer to

decide beforehand, whether, in the particular case he is to deal with, there is likely to be time and trouble saved by working according to the special rule, or by the ordinary method of common arithmetic. The last example above, for instance, may be quite as readily worked out by this ordinary method as by any special rule: thus:—

	£.	s.	
	56	12	Prime cost.
	11	4	Profit.
23)	67	16 (£2	
	46		
	21		
	20		
	436 (19s.		
	23		
	206		
	207		
	— 1		

Selling price £2 19s., less $\frac{1}{23}$s. = $\frac{12}{23}$d. = $\frac{1}{2}$d.

*** Of course, $\frac{12}{23}$d. could not be estimated at other than $\frac{1}{2}$d.

If another shilling be added to the profit, the division would leave no remainder; so that for the profit to be £11 5s. the selling price must be exactly £2 19s. per ton.

We shall now proceed to apply the preceding rules to mercantile and commercial transactions of a specific character; as also to those mechanical or handicraft employments in which calculation is necessary in order to estimate correctly the value of the work done, or of the materials supplied. In these applications, we may sometimes repeat, in substance, a rule of wider import, already given in the preceding pages. It may be convenient that this should now and then be done, rather than that the computer should continually have to turn to back references.

Calculations useful in the Wool Trade.

In this trade the weights used, although bearing the same names, are not always the same, as to the number of pounds represented by them, in different parts of the country. In some places the stone, usually 14 lbs., is a weight of 15 lbs.; in other places, a weight of 16 lbs.; and the weights of higher denominations—multiples of the stone—vary of course in like proportion. The usual weights for wool are as follows:—

TABLE OF WEIGHTS FOR WOOL.

7 lbs.	make 1 clove.	6 tod 1 stone make 1 wey	= 182. lbs.
2 cloves (or 14 lbs.)	„ 1 stone.	2 weys „ 1 sack	= 364.
2 stone (or 28 lbs.)	„ 1 tod.	12 sacks „ 1 last	= 4368.

In some places the weights are these, namely.

15 lbs.	make 1 stone.	8 tod (or 240 lbs.)	make 1 pack.
2 stone (or 30 lbs.)	„ 1 tod.	In Ireland, 16 lbs.	„ 1 stone.

Since a pack of wool weighs 240 lbs., it follows that at 15 lbs. to the stone, it weighs exactly 16 stone; at 16 lbs. to the stone, it weighs exactly 15 stone; but at 14 lbs. to the stone, it weighs 17 stone 2 lbs.

PROBLEM 1.

The price of 1 *lb. being given in pence, to find the price of a stone of* 14 *lbs., of* 15 *lbs., or of* 16 *lbs.*

RULE I.—Regard the pence as so many shillings; to this sum add $\frac{1}{6}$th of itself, if the stone be 14 lbs.; $\frac{1}{4}$th of itself, if the stone be 15 lbs.; and $\frac{1}{3}$rd, if it be 16 lbs.

For, by regarding the pence as so many shillings, you, in effect, multiply the price of 1 lb. by 12; that is, you get the value of 12 lbs.; and $\frac{1}{6}$th of this must be the value of 2 lbs.; $\frac{1}{4}$th, the value of 3 lbs.; and $\frac{1}{3}$rd, the value of 4 lbs.; so that these portions severally added to the price of 12 lbs., must give the price of 14 lbs., 15 lbs., and 16 lbs., respectively, or,

RULE II.—Having taken the price of 1 lb., in pence, as so many shillings, add to this amount the price of 2 lbs., 3 lbs., or 4 lbs., according as the stone is 14 lbs., 15 lbs., or 16 lbs.

EXAMPLES. (Stone of 14 lbs.)

1. If 1 lb. of wool cost 17*d.*, what will a stone of 14 lbs. cost?

	s.	*d.*				*s.*	*d.*
By Rule I. 6)	17	0	= cost of 12 lbs.	By Rule II.		17	0
	2	10	„ 2 lbs.	1*s.* 5*d.* × 2 =		2	10
Ans.	19*s.*	10*d.*	„ 14 lbs.		*Ans.*	19*s.*	10*d.*

2. If 1 lb. cost $23\frac{1}{2}d.$, what will a stone of 14 lbs. cost?

This example requires merely that we calculate the price at $2s.$,—which would be $28s.$ per stone, and then subtract 14 times $\frac{1}{2}d.$, or $7d.$, leaving for answer $27s.$ $5d.$

3. If 1 lb. cost $3s.$ $9\frac{1}{2}d.$, what will a stone cost?

The price, in pence, is $45\frac{1}{2}d.$; and regarding every penny as a shilling, we have $45\frac{1}{2}s.$, that is, £2 $5s.$ $6d.$ for the price of 12 lbs. therefore,

	£	s.	d.				£	s.	d.
6)	2	5	6	= Price of 12 lbs.		Or,	2	5	6
		7	7	= „ 2 lbs.		$3s.$ $9\frac{1}{2}d.$ × 2 =		7	7
	£2	$13s.$	$1d.$	= „ 14 lbs.			£2	$13s.$	$1d.$

4. If 1 lb. cost $5s.$ $6\frac{1}{4}d.$, what will a stone cost? *Ans.* £3 $17s.$ $3\frac{1}{2}d.$
5. If 1 lb. cost $1s.$ $10\frac{3}{4}d.$, what will a stone cost? *Ans.* £1 $16s.$ $6\frac{1}{2}d.$

Stone of 15 lbs.

6. What will a stone of 15 lbs. cost, at $2s.$ $1\frac{3}{4}d.$ per lb.?

The price per stone in pence is $25\frac{3}{4}d.$; and $25\frac{3}{4}s.$ = £1 $5s.$ $9d.$, the cost of 12 lbs.

4) £1	$5s.$	$9d.$	Or,	£1	$5s.$	$9d.$	=	cost of 12 lbs.
	6	$5\frac{1}{4}$	$2s.$ $1\frac{3}{4}d.$ × 3 =		6	$5\frac{1}{4}$	=	„ 3 lbs.
Ans. £1	$12s.$	$2\frac{1}{4}d.$		£1	$12s.$	$2\frac{1}{4}d.$	=	„ 15 lbs.

7. If 1 lb. cost $13\frac{1}{2}d.$, what will 15 lbs. cost? *Ans.* $16s.$ $10\frac{1}{2}d.$
8. If 1 lb. cost $2s.$ $7\frac{1}{4}d.$, what will 7 stone cost? *Ans.* £13 $13s.$ $5\frac{1}{4}d.$
9. If 1 lb. cost $1s.$ $9\frac{1}{2}d.$, what will 9 stone cost? *Ans.* £12 $1s.$ $10\frac{1}{2}d.$

Stone of 16 lbs.

10. If 1 lb. cost $2s.$ $5\frac{1}{2}d.$, what will a stone of 16 lbs. cost?

The price per stone, in pence, is $29\frac{1}{2}d.$; and $29\frac{1}{2}s.$ = £1 $9s.$ $6d.$, the cost of 12 lbs.

3) £1	$9s.$	$6d.$	Or,	£1	$9s.$	$6d.$	=	cost of 12 lbs.
	9	10	$2s.$ $5\frac{1}{2}d.$ × 4 =		9	10	=	„ 4 lbs.
Ans. £1	$19s.$	$4d.$		£1	$19s.$	$4d.$	=	„ 16 lbs.

11. If 1 lb. cost $17\frac{3}{4}d.$, what will a stone of 16 lbs. cost? *Ans.* £1 $3s.$ $8d.$
12. If 1 lb. cost $13\frac{1}{2}d.$, what will a stone cost? *Ans.* $18s.$
13. If 1 lb. cost $16\frac{1}{4}d.$, what will 3 stone cost? *Ans.* £3 $5s.$

The reverse problem, namely: Having the price of a stone, to find the price of 1 lb., may be readily enough *worked by* common division. For example:—

At £2 13*s.* 1*d.* per stone of 14 lbs., what is the cost of 1 lb.

```
14) 53s. 1d. (3s. 9½d. Ans.
    42
    —
    11
    12
   ———
    133 (9d.
    126
   ———
     7d. = 28f. (2 farthings.
```

Problem 2.

The price of 1 *lb. being given in pence, to find the price of a pack.*

Rule.—Regard the pence as so many pounds (£), and you will have at once the price of the pack.

For there are 240*d.* in £1, and 240 lbs. in one pack; so that by changing the pence, in the price of 1 lb., into so many pounds, we in effect multiply the price of 1 lb. by 240, and thus get the price of 240 lbs.

Examples.

1. The price of a pack of wool, at 15¾*d.* per lb., is £15¾ = £15 15*s.*
2. The price of a pack of wool, at 17¼*d.* per lb., is £17¼ = £17 5*s.*
3. The price of 7 packs, at 23¾*d.* per lb., is £166 5*s.*

Problem 3. (Converse of Prob. 2.)

The price of a pack of wool being given in pounds (£), *to find the price of* 1 *lb., or of* 1 *stone, clove, &c.*

Rule.—As many pounds as a pack costs, so many pence will 1 lb. cost, as is obvious.

Examples.

1. If a pack of wool cost £15 15*s.*, what is that per lb., and per clove?

£15 15*s.* = £15¾; therefore, the price per lb. is 15¾*d.* = 1*s.* 3¾*d.*; and the price per clove is 7 times this; namely, 9*s.* 2¼*d.*

2. If a pack of wool cost £17 5*s.*, what is the cost per stone (14 lb.)?

£17 5*s.* = £17¼; therefore, the price per lb. is 17¼*d.* = 1*s.* 5¼*d.*;

and the price per stone, 14 times this, is = { 17*s.* 3*d.* / 2 10½

(Prob. 1, p. 77.) Hence the price of a stone is = £1 0 1½*d.*

3. If a pack of wool cost £23 15*s.*, what would be the cost of a stone of 15 lbs.?

£23 15*s.* = £23¾; therefore, the price per lb. is 23¾*d.* = 1*s.* 11¾*d.*;

and the price per stone (15 lbs.) is (Prob. 1, p. 77) = { £1 3*s.* 9*d.* / 5 11¼

Ans. £1 9*s.* 8¼*d.*

or, since the price of 1 lb. is 2*s.* all but ¼*d.*, therefore the price of 15 lbs. is 30*s.* less 3¾*d.* = 29*s.* 8¼*d.*

4. If a pack of wool cost £25 15*s.* what will 1 lb. cost? *Ans.* 2*s.* 1¾*d.*

5. If a pack of wool cost £16 5*s.*, what will a stone of 16 lbs. cost? *Ans.* £1 1*s.* 8*d.*

Calculations in reference to Wheat, Oats, &c.

In order to find the price of any number of bushels, quarters, &c., from the price of one bushel, quarter, &c., being known, the direct way is to multiply the price of one by the number. This is the method of common arithmetic, and in certain cases is the most convenient method; as, for instance, when the number (or multiplier) is below 12. For higher multipliers, special rules, for arriving at the result more expeditiously, have been given in the preceding pages: the application of these rules is still further seen in the following examples.

[Although, in working these examples, we have usually referred to the rules conformably to which the operations have been performed, yet such formal reference will but seldom be necessary if the computer bear in mind, 1st, that whenever we reckon the articles by the dozen, we take the price of the single article in pence, and then replace the pence by that number of shillings: we thus get the price of 1 dozen. 2nd. Whenever we reckon articles by the *score,* we take the price of the single article in shillings,

and replace these shillings by so many pounds: we thus get the price of 1 score. 3rd. Whenever the articles are reckoned by the 240, we take the price of the single article in pence, and replace the pence by pounds, and thus get the price of 240. It will, however, be sufficient, in all cases, to regard the given number of articles, in each case, as consisting of dozens, with part of a dozen over; or of scores, with part of a score over; the articles, over and above the complete dozens or scores, to be computed for separately. In numbers higher than the number 20, it is optional whether we compute by the dozen or by the score.]

EXAMPLES.

1. What is the cost of 18 bushels of corn, at 5*s.* 4*d.* per bushel?

As 18 is 12 + 6, we first calculate the price of a dozen (p. 55), and then add half that price for the 6. Thus:—Taking 64*d.* as 64*s.*, and adding the half to it, we have 96*s.* = £4 16*s.*, the cost required. But the work would be easy enough by the ordinary method: thus, 5*s.*, × 18 = 90*s.*, to which adding a third of 18*s.* for the 4*d.*, the result is 96*s.*

2. 23 quarters, at £2 9*s.* 3*d.* per quarter?

Here expressing the price in shillings, and then counting these as pounds (Rule, p. 62), we have £49¼ =

	£	s.	d.	
	£49	5*s.*	0*d.*	for 20 qrs.
For 3 qrs.	7	7	9	
Price of 23 qrs.	£56	12*s.*	9*d.*	

3. 26 bushels, at 4*s.* 10½*d.* per bushel? [4*s.* 10½*d.* = 58½*d.*]

	£	s.	d.
Price of 12 bush. =	2	18	6
„ „ „ =	2	18	6
„ 2 „ =		9	9
	£6	6*s.*	9*d.*

Or thus: (See Table, p. 50)
4*s.* 10½*d.* = 4⅞*s.*; and

£4⅞ =	£4	17*s.*	6*d.*	= price of	20
also	1	9	3	= „	6
	£6	6*s.*	9*d.*	= „	26

4. 30 quarters of barley, at 27*s.* 9*d.* per qr.? [27*s.* 9*d.* = 27¾*s.*]

	£	s.	d.	
£27¾ =	27	15	0	= price of 20 (Rule, p. 62.)
One half =	13	17	6	= „ 10
	£41	12*s.*	6*d.*	= „ 30

5. 23 quarters, at 29*s*. 5*d*. per quarter? [29*s*. 5*d*. = $29\frac{1}{3}$*s*. + 1*d*.]

	£	s.	d.	
£$29\frac{1}{3}$ =	29	6	8	= price of 20, at 29*s*. 4*d*.
	4	8	0	= „ 3 „
		1	11	= „ 23 at 1*d*.
	£33	16*s*.	7*d*.	= „ 23 „ 29*s*. 5*d*.

Or thus: 24, at 353 pence, amount to the same as 353 at 24*d*., or 2*s*.; that is, to 706*s*. = £35 6*s*. 0*d*.
from which take 1 9 5

There remains £33 16*s*. 7*d*.

6. 37 quarters, at 25*s*. 10*d*. per quarter? [25*s*. 10*d*. = $25\frac{3}{4}$*s*. + 1*d*.]

	£	s.	d.	
£$25\frac{3}{4}$ =	25	15	0	= price of 20, at 25*s*. 9*d*.
	12	17	6	= „ 10 „
	9	0	3	= „ 7 „
		3	1	= „ 37 at 1*d*.
	£47	15*s*.	10*d*.	= „ 37 at 25*s*. 10*d*.

Or thus: 36 at 310*d*. = 310 at 3*s*. = 930*s*. =

£	s.	d.	
46	10	0	= price of 36
1	5	10	= „ 1
£47	15*s*.	10*d*.	= „ 37

NOTE.—The last two examples may be otherwise worked, thus:

Ex. 5. £$29\frac{5}{12}$ = £29 8*s*. 4*d*., price of 20
4 8 3 „ 3

£33 16*s*. 7*d*. „ 23

Ex. 6. 25*s*. 10*d*. = $25\frac{5}{6}$*s*. (See Table p. 50.)

	£	s.	d.	
£$25\frac{5}{6}$ =	25	16	8	= price of 20
	12	18	4	= „ 10
	9	0	10	= „ 7
	£47	15*s*.	10*d*.	= „ 37

7. 32 quarters of wheat, at £2 11*s.* 10*d.* per qr.?

The price per qr. here is $51\frac{3}{4}$*s.* + 1*d.*; therefore (Rule, p. 62)—

£$51\frac{3}{4}$ = £51	15*s.*	0*d.*	=	price of 20, at £2 11*s.* 9*d.*	
25	17	6	=	„ 10 „ „	
5	3	6	=	„ 2 „ „	
	2	8	=	„ 32 at 1*d.*	
£82	18*s.*	8*d.*	=	„ 32 at £2 11*s.* 10*d.*	

[This example may also be worked as in the note to Ex. 6, the price per quarter here being $51\frac{5}{6}$*s.*]

As already observed, it is not in every case that the special rule, for working examples really coming under that rule, has any marked advantage over the general rule, taught in books of arithmetic. The merchant or trader, accustomed to calculations peculiar to his own calling, will see, at a glance, whether, in any particular case that may come before him, the special or the general rule will be the more convenient for him to use. By way of comparison, we shall take the last three examples, and work them each by the general rule.

Ex. 5.

	£	*s.*	*d.*	
	1	9	5	× 3
			5	
	7	7	1	
			4	
Price of 20 =	£29	8	4	
„ 3 =	4	8	3	
„ 23	£33	16*s.*	7*d.*	

Ex. 6.

	£	*s.*	*d.*	
	1	5	10	× 1
			6	
	7	15	0	
			6	
Price of 36 =	46	10	0	
„ 1 =	1	5	10	
„ 37 =	£47	15*s.*	10*d.*	

Ex. 7.

£	*s.*	*d.*	
2	11	10	
		8	
20	14	8	
		4	
£82	18*s.*	8*d.*	= *Ans.*

The special rule has the more decided advantage when

the number to be computed for is an exact number of dozens, or scores, as in each of the examples following.

8. Required the price of 40 qrs. of oats, at 42*s.* 6*d.* per qr.?
   £$42\frac{1}{2} \times 2 =$ £85, *Ans.*
9. 60 qrs., at 52*s.* 9*d.* per qr.? £$52\frac{3}{4} \times 3 =$ £$158\frac{1}{4} =$ £158 5*s.*, *Ans.*
10. 80 qrs., at 54*s.* 6*d.* per qr.? £$54\frac{1}{2} \times 4 =$ £218., *Ans.*
11. 36 bushels, at 5*s.* 4*d.* per bushel? 64*s.* $\times\ 3 = 192$*s.* = £9 12*s.*, *Ans.*
12. 48 bushels, at 4*s.* $10\frac{1}{2}$*d.* per bushel?
    $58\frac{1}{2}$*s.* $\times\ 4 = 234$*s.* = £11 14*s.*, *Ans.*

NOTE.—5 bushels of corn is called a load of corn, but a cart-load is 40 bushels. The load has reference to porterage; the cart-load to horse carriage. It may also be noticed here, that the sack varies in measure with the commodities measured. When coals were measured by the bushel, 3 bushels went to the sack: in corn of all descriptions, 4 bushels (not heaped up, as with coals) make a sack; but of flour, there are 5 bushels to the sack. A quarter of corn, or of malt, is 2 sacks, that is 8 bushels, as already stated; and a cart-load is 10 sacks, or 5 quarters.

The following Table will be found useful to retail corn-dealers.

TABLE.

From the price of a Quart to find the price of a Bushel.

Quart.	Bushel.		Quart.	Bushel.		Quart.	Bushel.	
d.	*s.*	*d.*	*d.*	*s.*	*d.*	*d.*	*s.*	*d.*
$\frac{1}{4}$		8	2	5	4	$3\frac{3}{4}$	10	0
$\frac{1}{2}$	1	4	$2\frac{1}{4}$	6	0	4	10	8
$\frac{3}{4}$	2	0	$2\frac{1}{2}$	6	8	$4\frac{1}{4}$	11	4
1	2	8	$2\frac{3}{4}$	7	4	$4\frac{1}{2}$	12	0
$1\frac{1}{4}$	3	4	3	8	0	$4\frac{3}{4}$	12	8
$1\frac{1}{2}$	4	0	$3\frac{1}{4}$	8	8	5	13	4
$1\frac{3}{4}$	4	8	$3\frac{1}{2}$	9	4	$5\frac{1}{4}$	14	0

For every additional farthing in the price of a quart, another 8*d.* must be added to the price of the bushel; for there are 32 quarts (8 gallons) in a bushel, and 32 farthings make 8*d.* If the dealer purchase by the quarter (8 bushels), and retail by the bushel, then every farthing added to the bushel adds 2*d.* to the quarter; so that as many twopences as he proposes for his profit on the prime cost of the quarter, so many farthings must he add to the prime cost *of the bushel.* But the prime cost per quart, or per bushel,

may not be readily ascertainable; it may involve a fraction of a farthing: thus, in the Table above, if, for instance, instead of 5*s*. 4*d*., the corn were 5*s*. 6*d*. per bushel, the price of the quart would contain such a fraction. In this case, the profit, at 2¼*d*. per quart, would be 2*d*. less than 8*d*., that is, it would be 6*d*.; and at 2½*d*., it would be 6*s*. 8*d*. less 5*s*. 6*d*., that is, 1*s*. 2*d*., and so on.

Calculations in reference to Flour.

A sack of flour weighs 2½ cwt., or 20 stone, or 280 lbs., and as, in certain calculations with which the flour-factor and the retail dealer ought to be familiar, this number 280 and its subdivisions very frequently enter, it will facilitate the numerical work if the following particulars be preserved in the memory, namely 280*d*. = 23*s*. 4*d*.; 280 farthings = 5*s*. 10*d*., or 70*d*.; and 280 sevenths of a farthing = 10*d*.; so that 280 fourteenths of a farthing = 5*d*.

That these are relations useful to be borne in mind the reader may convince himself even now, at the outset, for he here perceives that at 1*d*. per lb., a sack of flour will cost £1 3*s*. 4*d*., and at a farthing per lb., 5*s*. 10*d*. He sees, moreover, that for every farthing added to the price of the lb., 5*s*. 10*d*. is added to the price of the sack; that every seventh of a farthing added to the price of the lb., adds 10*d*. to the price of the sack; and, consequently, that a whole farthing added to the price of seven lbs. (half a stone), adds 10*d*. to the price of the sack; while the additional farthing to the price of a stone increases the price of the sack by 5*d*. And thus the retailer of flour in small quantities may readily estimate his profit, per sack, upon every advance of one farthing in the price, per lb. or per stone.

We shall now give a few examples showing the application of such of the rules, in the foregoing pages, as are available in business transactions in flour, and shall then supply the additional rules which these special transactions require, and which, to the retail dealer in particular, will be found to be of essential service.

Examples in the Purchase of Flour.

1. What will 13 stone of flour come to, at 1*s*. 3*d*. per stone?

1*s*. 3*d*. = 15*d*., therefore (Rule, p. 77) 15*s*. + 1*s*. 3*d*. = 16*s*. 3*d*., *Ans.*

2. What will 14 stone of flour come to, at 1*s.* 4*d.* per stone?

The price of 2 stone is 2*s.* 8*d.*, therefore (Rule, p. 77) 16*s.* + 2*s.* 8*d.* = 18*s.* 8*d.*, *Ans.*

3. What will 30 sacks of flour cost, at 29*s.* 3*d.* per sack?

		£	s.	d.
By the Rule, p. 62, the price of 20 sacks is £29¼	=	29	5	0
" 10 "	=	14	12	6
30	=	£43	17*s.*	6*d.*

4. What is the price of 20 sacks of flour, at 27*s.* 9*d.* per sack? *Ans.* £27 15*s.*
5. What is the price of 60 sacks, at 32*s.* 3*d.* per sack? *Ans.* £96 15*s.*
6. What is the price of 80 sacks, at 35*s.* 10*d.* per sack? *Ans.* £143 6*s.* 8*d.*
7. What is the price of 38 sacks, at 34*s.* 8*d.* per sack? *Ans.* £65 17*s.* 4*d.*

PROBLEM 1.

The price of 1 *lb. of flour being given, to find the price of a sack.*

RULE I.—Multiply 5*s.* 10*d.* by the number of farthings in the price of 1 lb. Or,

RULE II.—Annex a cipher to the number of farthings in the price of 1 lb., and call the result pence. Multiply by 7, and the product will be the price of a sack.

EXAMPLES.

1. At 1¾*d.*, and 2¾*d.*, per lb., what will be the corresponding prices of a sack of flour?

By Rule I.

1. At 7 farthings per lb.: 5*s.* 10*d.*
   7
   Ans. 40*s.* 10*d.*

2. At 11 farthings: 5*s.* 10*d.*
   11
   Ans. 64*s.* 2*d.*

By Rule II.

1. At 7 farthings: 70*d.* = 5*s.* 10*d.*
   7
   Ans. 40*s.* 10*d.*

2. At 11 farthings: 110*d.* = 9*s.* 2*d.*
   7
   Ans. 64*s.* 2*d.*

The reason of Rule I. is pretty obvious: 5*s*. 10*d*. is the price of a sack at a farthing per lb.; consequently, at any number of farthings per lb., the price of a sack must be 5*s*. 10*d*. multiplied by that number.

Rule II. is explained thus: By annexing a cipher to the price in farthings, we, in effect, multiply that price by 10, and by calling the result pence, we virtually again multiply by 4; that is, the price is now multiplied by 40; and then the final multiplication by 7 completes the multiplication of the price of 1 lb. by the number 280, and thus gives the price of 280 lbs. This second rule is a little the more convenient of the two when the number of farthings in the price of 1 lb. exceeds 12.

2. What is the price of a sack of flour, at $3\frac{1}{4}$*d*. per lb.?
At 13*f*., by Rule 2, 130*d*. = 10*s*. 10*d*.; and this × 7 = 75*s*. 10*d*.
3. What is the price of a sack of flour, at $2\frac{1}{4}$*d*. per lb.? *Ans*. 52*s*. 6*d*.
4. What is the price of a sack of flour, at $3\frac{1}{2}$*d*. per lb.? *Ans*. 81*s*. 8*d*.

Problem 2. (Converse of Prob. 1.)

The price of a sack of flour being given, to find the price of 1 *lb.*

Rule.—Divide the price of the sack, in pence, by 70; the result will be the number of farthings per lb.

Examples.

1. A sack of flour costs 40*s*. 10*d*., another costs 64*s*. 2*d*.; required the respective prices per lb.?

One sack costs 490 pence, the other 770 pence; hence by the Rule:

$$7{,}0)\ 49{,}0 \qquad\qquad 7{,}0)\ 77{,}0$$

$$7f. = 1\tfrac{3}{4}d.\ \text{per lb.} \qquad\qquad 11f. = 2\tfrac{3}{4}d.\ \text{per lb.}$$

The truth of the above rule will be seen from the following considerations. The price of 1 lb., in farthings, will be obtained by dividing the price, in farthings, of a sack, by 280. But this is the same as dividing successively by 4 and by 70. Now by regarding the pence-price of a sack as so many farthings, we in reality take a fourth part of that price, or divide it by 4; and then again by dividing by 70, the result must be the same number of farthings

that we should get by dividing at once by 280. And this is the Rule.

The price of a sack may, no doubt, be such that the 280th part of it may not be an exact number of farthings; in such a case, a fraction of a farthing will be unavoidable, as in the example next following, namely:—

2. If a sack of flour cost 41*s.* 10*d.*, what does it cost per lb.?

Here the price, in pence, is 502*d.*, and $502 \div 70 = 7\frac{12}{70}$? so that the accurate price of 1 lb. is 7 farthings and 12 seventieths, that is 6 thirty-fifths of a farthing besides. Such a price could not be paid in existing coin. But if 12 pence were taken from the cost of the sack (12 being the remainder, or overplus, after the division by 70), that is, if the price per sack were reduced to 40*s.* 10*d.*, then the price per lb. would be exactly 7 farthings. The retail dealer, knowing this, can readily find his exact profit per sack by increasing the 7 farthings by 1 farthing, then by another farthing, and so on; every additional farthing to the price of a lb. adding 5*s.* 10*d.* to the selling price of the sack: thus, at 8 farthings, or 2*d.*, per lb., the profit on the sack, at 41*s.* 10*d.*, would be 4*s.* 10*d.*: at $2\frac{1}{4}$*d.* per lb., the profit would be 10*s.* 8*d.*, and so on.

PROBLEM 3.

The price of a stone of flour being given, to find the price of a sack.

RULE.—The price of a stone, in farthings (regarded as so many pence), multiplied by 5, will give the price of a sack.

For by regarding the number of farthings, in the price of a stone, as so many pence, and then multiplying by 5, we, in effect, multiply the price by 4, and the product by 5; that is, we multiply by 20.

NOTE.—If the price of the stone be in shillings, without any pence, then the price of the sack is just so many pounds, as is obvious, and no calculation is required; but if the price of the stone comprise shillings and pence, then, instead of turning the shillings into farthings, it is better to write, for these shillings, so many pounds, and to deal as the rule directs with the odd pence only.

EXAMPLES.

1. If a stone of flour cost $15\frac{3}{4}$*d.*, what will a sack cost?

$15\frac{3}{4}d. = 63f.$; and $63d. = 5s.\ 3d.$, which $\times\ 5 = 26s.\ 3d.$, *Ans.*

Or (NOTE above), $3\frac{3}{4}d. = 15$ farthings; and $15d. = 1s.\ 3d.$, which $\times\ 5 = 6s.\ 3d.$; and prefixing to this £1, for the shilling, we *have £1 6s. 3d.* for the price of the sack.

2. If a stone of flour cost $11\frac{1}{4}d.$, what will a sack cost?
   $11\frac{1}{4}d. = 45f.$; and $45d. = 3s.\ 9d.$, which $\times\ 5 = 18s.\ 9d.$, *Ans.*
3. If a stone of flour cost $13\frac{1}{2}d.$, what will a sack cost?
   (Note, p. 88). *Ans.* £1 2*s.* 6*d.*
4. If a stone of flour cost 1*s.* $7\frac{1}{2}d.$, what will a sack cost?
   (Note, p. 88). *Ans.* £1 12*s.* 6*d.*

[The last two examples may be worked by aid of the Table (p. 50), thus: Ex. 3. $13\frac{1}{2}d. = 1\frac{1}{8}s.$; and $£1\frac{1}{8} = £1\ 2s.\ 6d.$ Ex. 4. $1s.\ 7\frac{1}{2}d. = 1\frac{5}{8}s.$; and $£1\frac{5}{8} = £1\ 12s.\ 6d.$ But reference to a Table for such particulars may always be rendered unnecessary by a very trifling amount of calculation: thus, in the present case,—

$$1\tfrac{1}{2}d. = \frac{1\frac{1}{2}}{12}s. = \frac{3}{24}s. = \frac{1}{8}s.\text{; and } 7\tfrac{1}{2}d. = \frac{7\frac{1}{2}}{12}s. = \frac{15}{24}s. = \frac{5}{8}s.$$

Also $£\frac{1}{8} = \frac{20}{8}s. = 2s.\ 6d.$; and $£\frac{5}{8} = \frac{100}{8}s. = 12s.\ 6d.$ And similarly in all cases.]

Problem 4. (Converse of Prob. 3.)

The price of a sack of flour being given, to find the price of a stone.

Rule.—Divide the number of pence in the price of a sack (regarding these pence as so many farthings) by 5, and the result will be the price of a stone. For the price of a stone is

$$\frac{\text{pence in sack}}{20}d. = \frac{\text{pence in sack} \times 4}{20}f. = \frac{\text{pence in sack}}{5}f.,$$

which is the Rule.

Examples.

1. The price of a sack of flour being 26*s.* 3*d.*, required the price of a stone?
   $26s.\ 3d. = 315d.$; and $315f. \div 5 = 63f. = 15\frac{3}{4}d.$, *Ans.*
2. If a sack of flour cost 18*s.* 9*d.*, what will a stone cost?
   $18s.\ 9d. = 225d.$; and $225f. \div 5 = 45f. = 11\frac{1}{4}d.$, *Ans.*
3. If a sack of flour cost 22*s.* 6*d.*, what will a stone cost? *Ans.* $13\frac{1}{2}d.$
4. Required the cost of a stone of flour, at 32*s.* 6*d.* per sack?
   Ans. 1*s.* $7\frac{1}{2}d.$
5. If a sack of flour cost 24*s.*, at what price per stone must it be sold in order that the retailer may gain about 5*s.* profit on the sack?

[It may be instructive to work this example at length.]
The selling price must be at the rate of 29*s.* per sack: $29s. = 348d.$; and $348f. \div 5 = 69\frac{3}{5}f. = 17\frac{1}{4}d. + \frac{3}{5}f.$ This is the exact price at which a stone must be sold, in order that the gain, per sack, may be just 5*s.*

The gain would also be exactly this sum if the cost price of the sack were 3*d.* less than 24*s.*, and the selling price, per stone, 17¼*d.*; seeing that the remainder from the foregoing division by 5 is 3. Hence, disregarding the fraction ⅗*f.*, the gain, at 17¼*d.* per stone, is 5*s.* — 3*d.*, that is, 4*s.* 9*d.*; the addition of a farthing to this price would add 5*d.* to the price per sack, that is, at 17½*d.* per stone, the profit, at 24*s.* per sack, would be 5*s.* 2*d.*

As the remainder, arising from dividing the farthings by 5, is always so many fifths of a farthing, it is well to remember that one-fifth of a farthing added to the price of the stone, adds 1*d.* to the price of the sack; thus, the ⅗*f.* added to the 17¼*d.* per stone above, raises the price of the sack from 24*s.*—3*d.* to 24*s.*

Calculations respecting Carriage of Goods by Railway, &c.

The highest weight-denomination in the goods carried may be the cwt., or it may be the ton: special rules for the two cases here follow.

Problem 1.

To find the charge for carriage at a given sum per cwt.

Rule.—Call the number of cwts. so many shillings: we shall thus get the cost of carriage of these cwts., at the rate of 1*s.* per cwt.; and for the additional smaller weights the charge, at this rate, would be, for ¼ cwt., 3*d.*; for 14 lbs. (a stone), 1½*d.*; and for 7 lbs., ¾*d.* Add these several charges together, and multiply the amount by the number of shillings per cwt., taking parts for the odd pence if there be any, as in the following worked examples.

Note.—If the charge be pence only, without any shillings, we are still, as here directed, to compute the charge at 1*s.*, and then, taking parts for the pence, to add together these parts only. For instance, in Ex. 2, page 91, if the charge for carriage be only 7*d.* per cwt., instead of 1*s.* 7*d.*, the work would be as here annexed.

At 1*s.* . .	13*s.*	10½*d.*
,, 6*d.* . .	6	11¼
,, 1*d.* . .	1	2
,, 7*d.* . .	8*s.*	1¼*d.*

Examples.

1. What will the carriage of 17 cwt. 3 qrs. 21 lbs. come to, at 3*s.* per cwt.?

   17*s.* + 9*d.* + ¾*d.* = 17*s.* 11¼*d.*, the charge at 1*s.* per cwt.; and 17*s.* 11¼*d.* × 3 = £2 13*s.* 9¾*d.*, the charge at 3*s.*, *Ans.*

2. What will the carriage of 13 cwt. 3 qrs. 15 lbs. come to, at 1*s*. 7*d*. per cwt.?

[The 15 lbs. must be estimated as 1 stone (14 lbs.).]

Charge at 1*s*. per cwt. = 13*s*. + 9*d*. + 1½*d*.	=	13*s*.	10½*d*.; and
½ of this (for 6*d*.)	=	6	11¼
⅙ of this last (for 1*d*.)	=	1	2
	Ans.	21*s*.	11¾*d*.

The charge made in this case would, no doubt, be 22*s*.

If the cost of carriage had been 2*s*. 7*d*. per cwt., we should have computed exactly as above, and then, to the result here arrived at, should have added 13*s*. 10½*d*., making the charge 35*s*. 10¼*d*.; and if the charge had been 3*s*. 7*d*. per cwt., we should have added twice 13*s*. 10½*d*., instead of once that sum; and so on, since this sum is the charge at 1*s*. per cwt. But when the charge is two or more shillings, besides odd pence, a little time and trouble may be saved by writing above the total charge at 1*s*. as many times this charge as there are extra shillings, and then adding all the amounts up; thus, taking the charge for the foregoing weight of goods at 3*s*. 7*d*. per cwt., we should recommend working as in the margin; the 13*s*. 10½*d*. being found, as above, and written down first; twice this sum being then placed immediately above it, and the half immediately below it.

s.	*d*.
27	9
13	10½
6	11¼
1	2
49*s*.	8¾*d*.

[It is obvious that if, instead of the cost of carriage being so much per cwt., the cost of the goods themselves were at that rate, the foregoing method of computation would be equally applicable to the finding of the cost of the whole.]

Problem 2.

When the charge for carriage is by the ton.

Rule.—Call the tons so many pounds (£), and the cwts. so many shillings; then, as in last problem, allow as follows: for ¼ cwt., 3*d*.; for 14 lbs., 1½*d*.; and for 7 lbs., ¾*d*., and compute as before. [The Rule applies as well to the cost of the goods as to the carriage (see Ex. 3, p. 93).]

If the charge per ton be more than £1, multiply the cost at £1 by the number of pounds (£), taking parts for the

shillings and pence, as in last problem; but if it be less than £1, the parts for the shillings are, alone, to be added together.

Examples.

1. What is the charge for the carriage of 7 tons 5 cwts. 3 qrs. 14 lbs., from London to Manchester, at the rate of 25*s*. 3*d*. per ton?

	£	s.	d.
Charge for 7 tons, at £1	7	0	0
" 5 cwts. "		5	0
" 3 qrs. "			9
" 14 lbs. "			$1\frac{1}{2}$
Total charge at £1 per ton	£7	5	$10\frac{1}{2}$
For 5*s*. $= \frac{1}{4}$ £	1	16	$5\frac{1}{2}$
3*d*. $= \frac{1}{20}$ 5*s*.		1	10
Total charge at 25*s*. 3*d*.	£9	4	2

The work admits of contraction: it has been spread out, as in the above form, merely to exemplify the rule in full detail. It is pretty obvious, however, that we may write down the total charge at the rate of £1 per ton, at once, so that the last four lines only comprehend all the work that is really requisite.

[It is scarcely necessary to state, in proof of the rule, that since 20 cwts. = 1 ton, the charge for 1 cwt., at 20*s*. per ton, must be 1*s*. If in the above example the charge per ton had been £2 5*s*. 3*d*., or £3 5*s*. 3*d*., or &c., instead of £1 5*s*. 3*d*., we should have multiplied the total charge at £1 by the number of these additional pounds per ton; and, writing the product above the £7 5*s*. $10\frac{1}{2}d$., the work would have stood as here annexed, supposing the rate of carriage to have been £3 5*s*. 3*d*. per ton.]

£	s.	d.
14	11	9
7	5	$10\frac{1}{2}$
1	16	$5\frac{1}{2}$
	1	10
£23	15*s*.	11*d*.

2. What is the charge for the carriage of 9 tons 9 cwts. 1 qrs. 16 lbs. from York to Newcastle, at the rate of 17*s*. 6*d*. per ton?

By the Rule, the charge, at £1 per ton, is	£9	9*s*.	$4\frac{3}{4}d$.*
" 2*s*. 6*d*. $= \frac{1}{8}$ £	1	3	8 (*Subtract.*)
" 17*s*. 6*d*.	£8	5*s*.	$8\frac{3}{4}d$.†

* $\frac{1}{4}d$. being allowed for the 2 lbs. above 14 lbs.
† *This* would no doubt be regarded as £8 5*s*. 9*d*.

[The charge per ton here being less than £1 by 2*s*. 6*d*., the part for this deficiency is subtracted.]

3. 215 tons 17 cwts. 3 qrs. 9 lbs., at £9 11*s*. 6¼*d*. per ton?

	£	*s*.	*d*.	
	1727	2	8	(at £8 per ton.)
By the Rule, the charge, at £1 per ton, is	215	17	10	
" 10*s*. = ½ £ is	107	18	11	
" 1*s*. = $\frac{1}{10}$ 10*s*. is	10	15	10¾	
" 6*d*. = ½ 1*s*. is	5	7	11¼	
" ¼*d*. = $\frac{1}{24}$ 6*d*. is		4	6	= 108*d*. ÷ 2
Charge at £9 11*s*. 6¼*d*. =	£2067	7*s*.	8¾*d*.	

This would of course be considered as £2067 7*s*. 9*d*.

[In reference to the last item above, namely, 4*s*. 6*d*., it will be noticed that the 24th part of 108*s*. (which £5 7*s*. 11¼*d*. may be taken for) is the half of 108 pence.]

4. 45 tons 3 cwts. 3 qrs. 17 lbs. of goods were booked at Manchester for Edinburgh, at 1¼*d*. per ton per mile (distance 272 miles); the Lancashire Company carried the goods 80 miles; the York and Newcastle Company, 76 miles; the Newcastle and Berwick, 64 miles; and the Berwick and Edinburgh, 52 miles. What is the amount due to each Company?

The weight carried is 903 cwt. 101 lbs.

```
    903
   903
  903
    101
 ------
 101237 lbs.
      5 = No. of farthings per 2240 lbs. (1 ton.)
 ------
 506185 ÷ 2240 = 226f. = 56½d. = 4s. 8½d.
```

[It is plain that as many times as 2240 lbs. are contained in 5 times the whole weight in lbs., so many farthings will the carriage of that weight cost per mile. As, however, every number terminating in a 0 is divisible by 5, the number 2240 is divisible by 5, the quotient being 448; so that the multiplication by 5, as above, might have been dispensed with; it would have been sufficient merely to have divided 101237 by 448. And it is specially worthy of remembrance, not only that every number terminating in 0 is divisible by 5, but also that the quotient from this

division is always found by merely multiplying the number itself, after expunging the 0, by 2; thus, $2240 \div 5 = 224 \times 2 = 448$; $1370 \div 5 = 137 \times 2 = 274$; $63920 \div 5 = 6392 \times 2 = 12784$; and so on, generally; because expunging the 0 is dividing by 10, the result being just half what the division by 5 would be.]

Proceeding with the arithmetical work, we have now to calculate the separate charges at 4*s*. 8½*d*. per mile, first for 80 miles, then for 76 miles, then for 64 miles, and lastly for 52 miles. The work for the first of these charges may be computed as follows:—

	£	*s.*	*d.*
4 at £4 [that is, 80 at 4*s*.] =	16	0	0
8 at 6*s*. 8*d*. [that is, 80 at 8*d*.] =	2	13	4
½ at 6*s*. 8*d*. [that is, 80 at ½*d*.] =		3	4
Charge for the Lancashire Company	£18	16*s*.	8*d*.

Now the charge for 4 miles, being 4 times 4*s*. 8½*d*., is 18*s*. 10*d*., and the charge for 12 miles is 3 times this, or £2 16*s*. 6*d*. Subtracting therefore the first of these sums from £18 16*s*. 8*d*., the second of them from the remainder, and then again from the next remainder, the computation is continued thus:—

	£	*s.*	*d.*		
Charge for the Lancashire Company	18	16	8	For 80	miles.
(*Subtract*)		18	10	„ 4	„
„ York and Newcastle „	£17	17*s*.	10*d*.	„ 76	„
(*Subtract*)	2	16	6	„ 12	„
„ Newcastle and Berwick „	£15	1*s*.	4*d*.	„ 64	„
(*Subtract*)	2	16	6	„ 12	„
„ Berwick and Edinburgh „	£12	4*s*.	10*d*.	„ 52	„
And the charge for the entire distance is	£64	0*s*.	8*d*.	„ 272	„

[The foregoing example does not, in strictness, belong to the present problem, inasmuch as mileage is the most important part of the railway charge: but we have thought that an instance of this kind would be instructive to the reader, and we could find a no more appropriate place for it than under the head of the present division of our sub-

ject. We have worked it above, with all the detail that a novice might require; but in this, as also indeed in most of the worked examples in this book, there is a fulness of detail which, in actual business, would be for the most part suppressed. What in real practice would be but mentally supplied must appear before the eye in a printed book.

The above calculation will suffice to show the Goods-Manager, at a Railway Station, how to compute the charge for any weight of goods, at any price per ton, per mile. It will also be a guide for Ticket-Clerks, in cases where several Companies are concerned in the carriage of the same goods; and will likewise be of service to computers in the Clearance-House.]

5. What is the charge for the carriage of 37 tons 13 cwt. 2 qrs. 18 lbs. from Bristol to Liverpool, at the rate of £1 13*s.* 10*d.* per ton? *Ans.* £63 14*s.* 11½*d.*

6. Required the charge for 43 tons 1 cwt. 1 qr., from Paddington to Edinburgh, at the rate of £2 7*s.* 9*d.* per ton? *Ans.* £102 16*s.* 3*d.*

7. Required the charge for 84 tons 3 cwts. 3 qrs. 20 lbs., from Manchester to Aberdeen, at 52*s.* 9*d.* per ton? *Ans.* £222 1*s.* 4¼*d.*

Calculations Useful in the Spirit Trade.

[It will be found convenient in this business to recollect that 63 farthings make 1*s.* 3¾*d.*, and 63 halfpence 2*s.* 7½*d.*]

Problem 1.

From the price of a gallon, to find the price of a hogshead.

Rule I.—Multiply £3 3*s.* by the number of shillings in the price per gallon; 5*s.* 3*d.* by the number of pence; and 1*s.* 3¾*d.* by the number of farthings; and add the results.

Rule II.—Regard the price, in shillings, per gallon as so many pounds and parts of a pound, and multiply it by 3: multiply the price, as given, also by 3, and add the results.

Of course, there will always be as many shillings (and parts) in the latter product as there are pounds (and parts) in the former.

EXAMPLES.

1. If the price of a gallon be 9*s*. 7¾*d*., what is the price of a hogshead?

By Rule I.

£3 3*s*. × 9 =	£28	7*s*.		
5*s*. 3*d*. × 7 =	£ 1	16*s*.	9*d*.	
1*s*. 3¾*d*. × 3 =		3*s*.	11¼*d*.	
Ans.	£30	7*s*.	8¼*d*.	

By Rule II.

Take the price per gal. at 9*s*. 8*d*.

3 £9⅔ + 3 (9*s*. 8*d*.) =	£30 9*s*. 0*d*.
Subtract 63 far. =	1*s*. 3¾*d*.
Ans.	£30 7*s*. 8¼*d*.

[NOTE.—3 × £9⅔ = £29; and 3 (9*s*. 8*d*.) = 29*s*. The notation 3 (9*s*. 8*d*.) means 3 times 9*s*. 8*d*.]

The reason of Rule I. will appear evident from considering that £3 3*s*. = 63 shillings; 5*s*. 3*d*. = 63 pence; and 1*s*. 3¾*d*. = 63 farthings; and that the shillings in the price of a hogshead must be equal to 63 times the shillings in the price of a gallon; the pence in the price of the former, equal to 63 times the pence in the price of the latter; and the farthings in the price of the former, equal to 63 times the farthings in the price of the latter, because 63 gallons make 1 hogshead. In the above, each denomination in the price of a gallon—shillings, pence, and farthings—is multiplied by 63, and the products are added.

As to Rule II., it is sufficient to notice that by taking the shillings in the price of a gallon as so many pounds, and multiplying by 3, we get the price of three-score, that is, of 60 gallons; to which the price of 3 gallons being added, the amount must be the price of 63 gallons.

It may be observed here that, whatever be the capacity of the cask,—whether it contain 63, 54, 36, &c., gallons, the mode of computation is similar, as respects Rule I. Thus, suppose the cask contains 54 gallons, at 9*s*. 7¾*d*. per gallon; then to find the price of the whole, we have only to remember that 54*s*. = £2 14*s*.; 54*d*. = 4*s*. 6*d*.; and 54*f*. = 1*s*. 1½*d*., and to proceed, after the model above, as follows:—

	£	*s*.	*d*.
£2 14*s*. × 9 =	24	6	0
4*s*. 6*d*. × 7 =	1	11	6
1*s*. 1½*d*. × 3 =		3	4½
Ans.	£26	0*s*.	10½*d*.

For the first of the sums here added is 9 times 54*s.*, which is, of course, the same as 54 times 9*s.*; the second, is 7 times 54 pence, this being the same as 54 times 7 pence; and the third, 3 times 54 farthings is the same as 54 times 3 farthings; so that the total amount of these three separate sums must be 54 times 9*s.* 7¾*d.* And the work would be similar for any other number of gallons,—the rule being perfectly general and quite irrespective of all peculiarity as to the composition of the number proposed. The number 54, taken above, is employed solely for the purpose of illustration; it happens to be a number favourably constituted for computing by dozens, as it is composed of 9 half-dozens; so that the work might be perhaps more expeditiously performed as in the margin after replacing 9*s.* 7¾*d.* by 115*s.* 9*d.* (Rule p. 54.)

	115*s.*	9*d.*
		9
2)	1041	9
2,0)	52,0	10½
	£26 0*s.*	10½*d.*

It may be as well to add here that, although the present problem concerns gallons only, yet this first rule equally applies, whatever be the nature of the single article, and whatever be the number of articles. We shall now give a few unworked examples for exercise in the two rules above.

2. If the price of a gallon be 6*s.* 3*d.*, what is the price of a hogshead? *Ans.* £19 13*s.* 9*d.*
3. If the price of a gallon be 14*s.* 9*d.*, what is the price of a hogshead? *Ans.* £46 9*s.* 3*d.*
4. If a gallon cost 13*s.* 4½*d.*, what will a hogshead cost? *Ans.*, £42 2*s.* 7½*d.*
5. If a gallon cost 11*s.* 5¾*d.*, what will 76 gallons cost? *Ans.* £43 12*s.* 5*d.*

NOTE.—As a tun is equal to four hogsheads, the price of a tun is found by multiplying the price of a hogshead by 4.

PROBLEM 2. (CONVERSE OF PROB. 1.)

From the price of a hogshead to find the price of a gallon.

The most convenient way of solving this problem is to proceed by the method of common arithmetic, namely:

RULE.—Divide the price of the hogshead by 63 (or by 7 and 9); the quotient will be the price of a gallon. For example:

If the price of a hogshead be £46 9*s*. 3*d*., what will be the price of a gallon?

```
      £   s.  d.                        £   s.  d.
      46  9   3          or,       7) 46   9   3
      20                              ----------
      —                            9)  6  12   9
63)  929 (14s. 9d. Ans.               ----------
     63                                14s. 9d. Ans.
    ----                              ----------
     299
     252
    ----
      47
      12
    ----
     567 (9d.
     567
    ----
```

As this is the process of ordinary arithmetic, additional examples may not be necessary; but if the reader desire further exercise in the present problem he can take the reverse of the examples in the problem preceding, just as we have here taken the reverse of Example 3.

The retailer of spirits can easily determine his profit upon the hogshead by recollecting that every penny added to the price of the gallon adds 5*s*. 3*d*. to the price of the hogshead. But the usual practice is to secure profit, not by increasing the cost price of the article, but by reducing its strength by diluting it. In this way the profit, per gallon, which accrues from adding a certain quantity of water, is not quite so readily estimated. Suppose, for example, that the prime cost per gallon is 12*s*., and that the retailer adds a pint of water to the gallon; then he has got 9 pints of the diluted spirits for 12*s*.; a gallon of it therefore (8 pints) costs him only eight-ninths of 12*s*., that is, 12*s*. less one-ninth of 12*s*.; so that, as $\frac{1}{9}$th of 12*s*. is 1*s*. 4*d*., this is the profit he receives upon every gallon of the mixture, provided he sell it at the cost price of the unreduced article.

The general principle may be expressed thus:—

Whatever fraction the water, added to the gallon of spirits, is of the whole mixture, that fraction of the cost price of the spirits is the profit upon a gallon of the mixture. Thus, if a pint of water be added to the gallon of spirits, the profit, per *gallon*, of the mixture, will be $\frac{1}{9}$th of the cost of the gallon

of spirits; if a quart be added, the profit, per gallon, will be $\frac{1}{5}$th the cost of the gallon of spirits; if a gallon be added, the profit, per gallon, of the mixture, will be half the cost of the gallon of spirits; and so on.

It may be useful to make this matter the subject of two or three distinct problems.

Problem 3.

The prime cost of a gallon of spirits being given, to find how much pure water must be added in order that an assigned amount of profit may be secured upon the outlay; the selling price, per gallon, being the same as the cost price of a gallon of the unadulterated article.

The water costs nothing, but when incorporated with the spirits it fetches the cost price of the spirits, per gallon. Hence, this cost price, multiplied by the fraction which the water is of a gallon, is the profit upon the whole mixture; and consequently the profit, divided by the price of a gallon of the spirits, must be equal to this fraction of a gallon of water. The rule is therefore this:—

Rule.—Divide the proposed profit by the given price per gallon; the quotient will be the fraction which expresses the portion of a gallon of water to be added.

Examples.

1. If a gallon of spirits cost 12*s*., how much water must be added to it in order that the mixture, at 12*s*. per gallon, may yield a profit of 2*s*.?

$$2s. \div 12s. = \tfrac{1}{6} \text{ of a gallon} = \tfrac{8}{6} \text{ pints} = 1\tfrac{1}{3} \text{ pints};$$

so that a pint and one-third of a pint of water must be added. This may be easily verified as follows: $\frac{1}{6}$ gallon of water being added, the measure of the mixture is $1\frac{1}{6}$ gallons; this, at 12*s*. per gallon, amounts to $12s. \times 1\frac{1}{6} = 14s.$, which gives a profit of 2*s*. upon the outlay per gallon.

2. If a gallon of spirits cost 13*s*. 6*d*., how much water must be added to secure a profit of 3*s*. upon this 13*s*. 6*d*.?
$3s. \div 13s.\ 6d. = 6 \div 27 = \frac{6}{27}$, the fraction of a gallon $= \frac{48}{27}$ pints $= 1$ pint and $\frac{21}{27}$ths of a pint, that is, $\frac{7}{9}$ths of a pint. We shall verify this result, as in the former example. The measure of the mixture (replacing $\frac{6}{27}$ by the equivalent fraction $\frac{2}{9}$) is $1\frac{2}{9}$ gallon, which, at 13*s*. 6*d*. per gallon, amounts to 27 sixpences $\times\ 1\frac{2}{9} =$

27 sixpences + 6 sixpences = 13*s.* 6*d.* + 3*s.*; so that the profit is 3*s.*

3. How much water must be added to a gallon of spirits at 15*s.* to secure a profit of 3*s.* 6*d.*? *Ans.* $\frac{7}{30}$ of a gallon.
4. How much water must be added to a gallon of spirits at 14*s.* 6*d.* to secure a profit of 2*s.* 9*d.*? *Ans.* $\frac{11}{58}$ of a gallon.
5. How much water must be added to a gallon of spirits at 18*s.* 4*d.* in order that the profit upon it may be 3*s.* 8*d.*? *Ans.* $\frac{1}{5}$ of a gallon.

[In the above problem the amount of profit is given to determine the quantity of water; but if the quantity of water be given to determine the amount of profit upon the sale of the whole mixture, then, whatever be the quantity of water, the profit will obviously be the worth of that quantity of unmixed spirits. How to find the profit per gallon of the mixture has been sufficiently explained in the directions immediately preceding the present problem.]

Problem 4.

1. *A given quantity of sweetened water being added to a gallon of spirits, to find the profit on the prime cost of the gallon, the selling price being at the rate of the cost price per gallon.*
2. *To find how much sweetened water must be added to a gallon of spirits in order that the profit upon that gallon may be a given sum, the selling price being the same as the cost price of the unadulterated spirits per gallon.*

The first part of this problem requires no special rule; it is plain that whatever be the quantity of sweetened water added, the profit will be the cost price of that quantity of pure spirits less the price of the sugar. For the second part of the problem we give the following rule:—

Rule.—Divide the proposed profit by the prime cost of the gallon of spirits diminished by the cost of the sugar used in sweetening a gallon of water; the quotient will be the quantity (or fraction of a gallon) of the sweetened water to be added.

For if a whole gallon of the sweetened water were to be added, the profit upon the mixture would be the cost price of a gallon of the pure spirits, diminished by the cost price of a gallon of the sweetened water; consequently whatever fraction of a gallon be added, the profit will be found by

multiplying the cost of a gallon of spirits less the cost of a gallon of the sweetened water, by that fraction; and therefore the fraction itself will be the quotient arising from dividing the proposed profit by the cost of a gallon of the spirits diminished by the cost of a gallon of the sweetened water; and this is the rule.

EXAMPLES.

1. If a gallon of spirits cost 12*s.*, how much sweetened water must be added to it, in order that the mixture, at 12*s.* per gallon, may yield a profit of 2*s.*, the cost of the sugar being 8*d.* for a gallon of the sweetened water?

$$2s. \div 11s.\ 4d. = 2s. \div 11\tfrac{1}{3}s. = \tfrac{6}{34} = \tfrac{3}{17} \text{ of a gallon.}$$

Or, reducing at once to fourpences, of which a shilling contains three, we have $6 \div 34 = \frac{3}{17}$, which is the portion of a gallon to be added to every gallon of pure spirits, in order that the profit on that gallon may be 2*s.* Or, 3 gallons (or 3 measures of any kind) of the sweetened water must be added to 17 gallons (or 17 like measures) of the spirits, in order that 2*s.* profit may be realized upon every gallon of pure spirits in the mixture. If the water were unsweetened, then to produce the same profit, per gallon of spirits, 3 gallons (or measures) of pure water must be added to 18 gallons (or measures) of spirits; or 1 of the former to 6 of the latter. (Ex. 1, Prob. 3.) As in the example here referred to, we shall now verify the foregoing conclusion.

The selling price of the whole $1\frac{3}{17}$ gallons is $12s. + \frac{3}{17}\ 12s.$, that is, $12s. + \frac{36}{17}s. = 12s. + 2s. + \frac{24}{17}d.$ And the cost price is $12s. + \frac{3}{17}\ 8d. = 12s. + \frac{24}{17}d.$; hence, subtracting this amount from the former, 2*s.* is the amount of profit on the purchased gallon.

2. If a gallon of spirits cost 13*s.* 6*d.*, how much sweetened water must be added to it so that the reduced spirits, at the same price per gallon, may yield a profit of 3*s.*; the cost of the sugar in a gallon of the sweetened water being 10*d.*?

   Ans. $\frac{9}{38}$ of a gallon; or 9 measures of the water to 38 of the spirits.

3. If a gallon of spirits cost 25*s.* 10*d.*, and a gallon of sweetened water cost 9*d.*, how much of the latter must be added to the former, so that the mixture, at 25*s.* 10*d.* per gallon, may yield a profit of 3*s.* 6*d.* upon the outlay?

   Ans. $\frac{6}{43}$ of a gallon; or 6 measures of the water to 43 of the spirits.

[It will of course be remembered, in all cases coming under the present problem, that the predetermined profit is that upon each gallon of pure spirits in the mixture, and not the profit upon each gallon of the mixture itself. To find this profit is the object of the next problem.]

Problem 5.

A given quantity of water, sweetened or unsweetened, being added to a gallon of spirits, to find the profit upon a gallon of the mixture if sold at as much per gallon as the undiluted article cost.

1. If pure water only be added to the gallon, the profit, upon the sale of the whole mixture, will obviously be equal to the cost of an equal measure of the undiluted spirits. Hence we have only to divide this profit upon the whole mixture by the number of gallons in it, in order to find the profit upon a single gallon of it.

2. If the water added be sweetened, the profit upon the whole mixture will be the cost of an equal measure of spirits less the cost of the sugar in the water added; so that, by dividing this profit by the number of gallons in the mixture, the result must be the profit on a single gallon of that mixture. Hence, whether the water be sweetened or unsweetened, the rule is as follows:—

Rule.—Find the profit upon the sale of the whole mixture, or take this profit if it be already assigned (Problems 3, 4), and divide it by the number of gallons in the whole; the quotient will be the profit per gallon on the diluted spirits.

Examples.

1. If a gallon of spirits cost 12*s.*, $\frac{1}{6}$ of a gallon of pure water must be added to it in order that the profit upon the whole mixture may be 2*s.* (Ex. 1, Prob. 3); what is the profit upon each gallon of the spirits so diluted?

Here the whole mixture measures $1\frac{1}{6}$ gallons, the profit on which is 2*s.*; therefore 2*s.* $\div$ $1\frac{1}{6}$, or $\frac{12}{7}$*s.* $=$ 1*s.* $8\frac{4}{7}$*d.* is the profit per gallon on the diluted spirits.

2. The quantity of pure water which must be added to a gallon of spirits, at 13*s.* 6*d.*, in order that the profit upon the whole mixture may be 3*s.*, is $\frac{2}{9}$ of a gallon (Ex. 2, Prob. 3); what is the profit, per gallon, on the diluted spirits? *Ans.* 2*s.* $5\frac{5}{11}$*d.*

3. The quantity of sweetened water, worth 10*d.* per gallon, to be added to a gallon of spirits, at 13*s.* 6*d.*, in order that the profit upon the whole mixture may be 3*s.*, is $\frac{9}{38}$ of a gallon (Ex. 2, Prob. 4); what is the profit, per gallon, on the diluted spirits? *Ans.* 2*s.* $5\frac{5}{47}$*d.*

[The reader is recommended to verify these two results;

that is, to prove first that if the profit upon a gallon of the mixture be 2*s.* $5\frac{5}{11}$*d.*, the profit upon $1\frac{2}{9}$ gallons will be 3*s.*; and secondly, if the profit upon a gallon of the mixture be 2*s.* $5\frac{5}{47}$*d.*, that the profit upon $1\frac{9}{38}$ gallons will be just the same, namely, 3*s.* It may be well to give a hint as to the best way of doing this. Take the latter case, but, to get rid of the fraction, multiply $1\frac{9}{38}$ by 38; we thus have 47; and 2*s.* $5\frac{5}{47}$*d.* $\times$ 47 $=$ 94*s.* $+$ 240*d.* $=$ 114*s.*, which divided by 38 gives 3*s.*, as it ought to do.]

Problem 6.

To find how much water, sweetened or unsweetened, must be added to each gallon of spirits, in order that the profit upon a gallon of the mixture may be an assigned sum.

Rule.—1. Add the proposed profit upon a gallon of the mixture to the cost of a gallon of the water. [If unsweetened this cost is of course nothing, and there is then to be no addition.]

2. Subtract the result from the cost of a gallon of the spirits, and divide the proposed profit by the remainder; the quotient will be the portion of a gallon of the diluting liquid to be added to each gallon of spirits.

Examples.

1. How much pure water must be added to a gallon of spirits, at 13*s.* 6*d.* in order that the profit upon the mixture may be at the rate of 2*s.* $5\frac{5}{11}$*d.* per gallon? We proceed by the Rule as follows:—

13*s.* 6*d.* $-$ 2*s.* $5\frac{5}{11}$*d.* $=$ 162*d.* $-$ $29\frac{5}{11}$*d.* $=$ $132\frac{6}{11}$*d.* We have therefore to execute the division of $29\frac{5}{11}$ by $132\frac{6}{11}$, and in order to accomplish this, without the fractions, we multiply both dividend and divisor by 11; we thus have, 324 $\div$ 1458, or $\frac{324}{1458} = \frac{2}{9}$ of a gallon of water, as we otherwise know it ought to be (see Ex. 2, p. 99). The $\frac{2}{9}$ is the fraction $\frac{324}{1458}$ in its lowest terms; for both numerator and denominator of that fraction are divisible by 162, the numerator giving 2 for quotient, and the denominator giving 9.

In this example the water is unsweetened, and therefore costs nothing. We shall now work an example in which the water is sweetened, and therefore involves an outlay; we shall then give the reasoning from which the rule is deduced.

2. How much sweetened water, worth 10*d.* a gallon, must be added to each gallon of spirits, at 13*s.* 6*d.*, in order that the profit upon the mixture may be at the rate of 2*s.* $5\frac{5}{47}$*d.* per gallon?

2s. $5\frac{5}{47}d. + 10d. = 3s. 3\frac{5}{47}d. = 39\frac{5}{47}d.$; then, by the Rule, $162d. - 39\frac{5}{47}d. = 122\frac{42}{47}d.$: and by this the profit per gallon, namely, $29\frac{5}{47}d.$, is to be divided. In order to readily perform the division, take 47 times dividend and divisor: we shall then have $1368 \div 5776$; or, dividing each number by 152, the fraction is $\frac{9}{38}$, the portion of a gallon of sweetened water to be added to each gallon of spirits. (See Ex. 3, p 102.)

The foregoing rule is deduced from the following considerations, to which it is necessary, for the clear perception of its truth, that the reader give careful attention.

1. The profit upon the whole mixture consists entirely of the profit upon the measure of diluting liquid which is added to the gallon of spirits, it being sold at the same price as an equal measure of the spirits costs.

2. Suppose, for convenience, we represent the unknown fraction of a gallon which measures the quantity of the diluting liquid to be added to the gallon of spirits, by the symbol q.;* then this portion of a gallon sells for the qth part of the cost of a gallon of the spirits; that is it sells for the cost of a gallon of the spirits multiplied by q, whatever fraction q may represent; but it costs the qth part of the cost of a gallon of the diluting liquid; that is, it costs the value of a gallon of the diluting liquid multiplied by the fraction q. Hence, the profit upon the whole mixture is what remains after subtracting this product from the former.

3. But we want to know the profit, not upon the whole mixture, but upon only a gallon of it; let us then imagine the excess above a gallon to be taken away; we shall thus take away from the profit on the whole the profit on the portion abstracted; and since the quantity of the mixture subtracted is just equal to the quantity of the diluting liquid that was previously added, the profit thus taken away from the whole profit is a qth part of the profit per gallon, and the remainder is the profit on the remaining gallon of the mixture. The qth part of the profit, per gallon, is, of course, expressed by multiplying the profit upon the whole gallon by the fraction q.

* The reader may replace the symbol q by any fraction he please, say, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{7}$, &c.; and then, substituting this fraction throughout the *following reasoning* for the letter q, he will find, *choose whatever fraction he may, that* the inference is the same.

4. It thus appears that the profit upon this remaining gallon of the mixture is to be found by subtracting from the cost of a gallon of the spirits when multiplied by q, the cost of a gallon of the diluting liquid when multiplied by q, and then further subtracting the profit, per gallon, of the mixture when multiplied by q:—each item is to be multiplied by q.

The inference therefore is, that if we add the profit upon a gallon of the mixture to the cost of a gallon of the diluting liquid, subtract the sum from the cost of a gallon of the spirits and multiply the remainder by the fraction q, the product will be the profit, per gallon, on the mixture.

5. But if the product of two factors be divided by either factor, the quotient must be the other factor. Hence, if we divide the profit, per gallon, on the mixture, by the above-mentioned remainder, the quotient will be the fraction q; that is, the fraction of a gallon of the diluting liquid which must be added to a gallon of the pure spirits, in order that the assigned profit, on a gallon of the mixture, may be realized. And hence the rule.

3. How much pure water must be added to a gallon of spirits, at 12*s.* in order that the profit upon a gallon of the mixture may be 1*s.* 8$\frac{4}{7}$*d.*?

Ans. $\frac{1}{6}$ of a gallon; or 1 gallon of water to 6 gallons of spirits.

4. How much sweetened water, worth 9*d.* per gallon, must be added to a gallon of spirits, at 15*s.* 9*d.*, in order that the profit, per gallon, on the mixture may be 2*s.* 6*d.*?

Ans. $\frac{1}{5}$ of a gallon; or 1 gallon of the water to 5 gallons of the spirits.

This problem may be otherwise investigated as follows. Suppose q gallons to be the quantity of water, sweetened or unsweetened, to be added; then it is plain that q (cost of a gallon of spirits $-$ cost of a gallon of water) $=$ profit on the $1 + q$ gallons; therefore, profit per

gallon $= \frac{q}{1+q}$ (cost of a gallon of spirits $-$ cost of a gallon of water);

therefore, $\frac{1+q}{q}$ profit per gallon $=$ cost of a gallon of spirits $-$ cost of a gallon of water;

therefore, $\frac{1}{q}$ profit per gallon $=$ cost of a gallon of spirits $-$ (cost of a gallon of water $+$ profit per gallon).

therefore $q =$

$$\frac{\text{Profit per gallon of mixture.}}{\text{cost of gal. of spirits} - (\text{cost of gal. of water} + \text{profit per gal. of mixture}).}$$

And this is the Rule at page 103.

But the Rule may be established in another and a more easy way, by aid of the general principle at page 118, as follows.

Let A be the number of shillings, or of pence, in the cost of a gallon of the spirits; C the number in the cost of a gallon of the water; and B, the number in the cost of a gallon of the mixture: also, let P denote the number of shillings, or pence, in the profit, per gallon, on this mixture. Then, by the principle referred to, a compound, at the stipulated price per gallon, will be obtained by mixing $B - C$ gallons of spirits with $A - B$ gallons of the water; that is, since $A - B = P$, and therefore $B = A - P$, and $B - C = A - C - P$,

$$A - (C + P) = \text{the number of gallons of spirits.}$$
$$\text{and } P = \quad ,, \quad \text{the water.}$$

Hence to one gallon of spirits there must be added $\dfrac{P}{A - (C + P)}$ gallons of the water, as above.

Problem 7.

To find the proportion in which spirits, at two different prices per gallon, must be mixed in order that the compound may cost an assigned intermediate price per gallon.

Rule.—Divide the difference between the higher price and the intermediate price by the difference between the intermediate price and the lower price; the quotient will be the quantity of the cheaper spirits to be added to a gallon of the dearer.

Or, divide the difference between the intermediate price and the lower price by the difference between the higher price and the intermediate price; the quotient will be the quantity of the dearer spirits to be added to a gallon of the cheaper.

Examples.

1. **What quantity of spirits, at 9*s*. 6*d*. per gallon, must be mixed with a gallon of spirits at 15*s*., so that the cost of the compound may be 13*s*. 6*d*. per gallon?**

15*s*. — 13*s*. 6*d*. = 1*s*. 6*d*. = 18*d*.: 13*s*. 6*d*. — 9*s*. 6*d*. = 4*s*. = 48*d*.; and $\frac{18}{48} = \frac{3}{8}$, the part of a gallon to be added; or 3 gallons of the inferior spirits to 8 of the superior, as may be thus verified: 3 gallons at 9*s*. 6*d*. cost 28*s*. 6*d*., and 8 gallons at 15*s*. cost 120*s*.; so that the entire 11 gallons in the compound cost 148*s*. 6*d*., and therefore the cost of a single gallon is 148*s*. 6*d*. ÷ 11 = 13*s*. 6*d*., as it ought to be.

2. What quantity of pure water must be added to a gallon of spirits worth 12*s*., in order that the mixture may cost at the rate of 10*s*. per gallon?

Here the inferior spirit (so to call it) is worth nothing. The first of the two differences is 12*s*. — 10*s*. = 2*s*.; the second is 10*s*. itself; therefore 2*s*. ÷ 10*s*. = $\frac{1}{5}$, the part of a gallon of water to be added. The measure of the whole mixture is therefore $1\frac{1}{5}$ gal., and its cost 12*s*.: hence its cost per gallon is 12*s*. ÷ $1\frac{1}{5}$, or 60*s*. ÷ 6 = 10*s*., as it ought to be. And there must be 1 gallon of water to every 5 gallons of spirits.*.

3. How much sweetened water, at 9*d*. per gallon, must be added to a gallon of spirits, at 15*s*. 9*d*., so that the compound may cost at the rate of 13*s*. 3*d*. per gallon?

15*s*. 9*d*. — 13*s*. 3*d*. = 2*s*. 6*d*., and 13*s*. 3*d*. — 9*d*. = 12*s*. 6*d*.
2*s*. 6*d*. ÷ 12*s*. 6*d*. = $2\frac{1}{2} \div 12\frac{1}{2} = \frac{5}{25} = \frac{1}{5}$ [See Ex. 4, Prob. 6.]

4. How much spirits, at 15*s*. per gallon, must be mixed with a gallon at 9*s*. 6*d*. in order that the compound may cost at the rate of 13*s*. 6*d*. per gallon?

13*s*. 6*d*. — 9*s*. 6*d*. = 4*s*.: 15*s*. — 13*s*. 6*d*. = 1*s*. 6*d*.: then

48*d*. ÷ 18*d*. = $\frac{48}{18} = \frac{8}{3} = 2\frac{2}{3}$ gallons, the quantity required. Let us verify this: the cost of the whole $3\frac{2}{3}$ gallons is 9*s*. 6*d*. + 15*s*. × $2\frac{2}{3}$ = 9*s*. 6*d*. + 40*s*. = 49*s*. 6*d*., and therefore the cost of 1 gallon is 49*s*. 6*d*. ÷ $3\frac{2}{3}$, or (multiplying each by 3), 148*s*. 6*d*. ÷ 11 = 13*s*. 6*d*., as it ought to be.

And in this way may the truth of the Rule be tested, and satisfactorily proved, in every individual example to which it is applied. To prove it generally, without reference to particular examples, like the proof of the preceding Rule (Prob. 6), would be rather a tedious business without the aid of Algebra. (See, however, p. 118.)

It is evident that the proportion in which the two kinds of spirits are to be mixed, so that the cost price of a gallon of the compound may be a predetermined sum, being found as in the foregoing examples, the retailer has only got to increase this cost price by what he proposes for profit, per gallon, of the mixture, in order to know the proper retail price per gallon.

As such calculations as the above, concerning the mixing of spirits, must be the same, whatever be the ingredients mixed, we shall continue the subject under a more general head.

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* It will be seen that the present Rule is but a more general form of that at p. 103 (Prob. 6). What here is inferior spirits, is there pure or sweetened water. A Rule still more general is given at p. 109.

Calculations useful in the Mixing of Teas, Sugars, Spirits, Grain, &c.

Problem 1.

When given measures (or weight) of ingredients, at different prices, are mixed together, to find the price of a single measure (or weight) of the mixture.

Rule.—Multiply the price of one measure (or weight) of each ingredient by the number of measures (or weight) in it, and add all the products together; add also the different measures (or weights) themselves together, and divide the former sum by the latter; the quotient will be the price of a single measure (or weight) of the mixture.

Examples.

1. If 4 cwt. of sugar, at 56*s.* per cwt., 7 cwt. at 43*s.*, and 5 cwt. at 37*s.*, be mingled together: what will 1 cwt. of the mixture be worth?

$$\begin{array}{rcl} s. & & s. \\ 56 \times 4 & = & 224 \\ 43 \times 7 & = & 301 \\ 37 \times 5 & = & 185 \\ \hline 16) & & 710s.\ (44\tfrac{3}{8}s. = 44s.\ 4\tfrac{1}{2}d.,\ \textit{Ans.} \end{array}$$

It is plain that in every case, as well as here, the worth of the whole mixture must be the worth of all the ingredients. In the present instance the worth of the whole 16 cwt. in the mixture is found to be 710*s.*; and therefore the worth of 1 cwt. of it must be the 16th part of 710*s.*, namely, 44*s.* 4½*d.*

2. If 27 bushels of wheat at 5*s.* 6*d.* per bushel, the same quantity of rye at 4*s.* per bushel, and 14 bushels of barley at 3*s.* per bushel, be mixed together, what will be the worth of a bushel of the mixture?

$$\begin{array}{lllcrl} 27 & \text{at } 5s. & 6d. & = & 148s. & 6d. \\ 27 & ,,\ 4 & 0 & = & 108 & 0 \\ 14 & ,,\ 3 & 0 & = & 42 & 0 \\ \hline 68) & & & & 298 & 6\ (4s.\ 4\tfrac{1}{2}d. + \tfrac{12}{17}f.,\ \textit{Ans.} \end{array}$$

If the mixture be sold at 4*s.* 5*d.* per bushel, the gain upon the whole 68 bushels would be 68 times 1 farthing plus 68 times $\frac{5}{17}f.$; that is, 68*f.* + 20*f.* = 88*f.* = 1*s.* 10*d.*

3. *If 5 gallons* of wine at 7*s.* per gallon, 9 gallons at 8*s.* 6*d.*, and 14¼ *gallons at 5s. 10d.* be mixed together, what will a gallon of the *mixture* be worth? *Ans.* 6*s.* 10½*d.*

4. If 20 bushels of wheat at 5*s.*, 36 bushels of rye at 3*s.*, and 40 bushels of barley at 2*s.* be mixed together, what will be the worth of a bushel of the mixture? *Ans.* 3*s.*

5. If 20 gallons at 5*s.* 4*d.*, 12 at 5*s.*, 30 at 6*s.*, and 20 at 4*s.* 6*d.* be mixed together, what will the mixture be worth per gallon?
Ans. 5*s.* $3\frac{3}{4}d. + \frac{25}{41}f.$

PROBLEM 2.

The price of each of the several ingredients being given, to find in what proportions they may be mixed together, in order that the composition may bear a given price.

RULE.—Write the prices of the ingredients one under another in a column, commencing with either the lowest price or the highest, and proceeding in order, leaving a small space to separate the prices, each of which is higher than the proposed intermediate price, from those which are lower; and to the left of the column, against this space, write the given intermediate price.

2. Link together, by a curve or connecting line, each number above the space with one or other of the numbers below it, and each number below the space with one or other of the numbers above it.

3. Against the first number in the column of numbers write the difference between the number with which it is linked and the isolated number to the left of the space; and if it be linked to more numbers than one, write the sum of all the differences; and do the same with every number in the column. The numbers thus written against the several prices will express what quantities, at those prices, may be mixed together.

NOTE.—When more than two ingredients are to be mixed, they may be mixed in different proportions, and yet be worth the assigned price per gallon, pound, cwt., &c., as will be sufficiently seen in the following examples.

EXAMPLES.

1. In what proportion may teas, at 4*s.* 8*d.*, 4*s.*, 3*s.* 8*d.*, and 2*s.* 6*d.* per lb., be mixed, in order that the mixture may be worth 3*s.* 10*d.* per lb?

			Proof.
	56	16 at 4*s.* 8*d.*	16 at 56*d.* = 896*d.*
	48	2 ,, 4*s.*	2 ,, 48 = 96
46			
	44	2 ,, 3*s.* 8*d.*	2 ,, 44 = 88
	30	10 ,, 2*s.* 6*d.*	10 ,, 30 = 300
			30) 1380 (46*d.* = 3*s.* 10*d.*

Otherwise,

				Proof.	
	56	2 at 4*s*. 8*d*.		2 at 56*d*.	= 112*d*.
	48	16 ,, 4*s*.		16 ,, 48	= 768
46					
	44	10 ,, 3*s*. 8*d*.		10 ,, 44	= 440
	30	2 ,, 2*s*. 6*d*.		2 ,, 30	= 60
				30)	1380 (46*d*.

Hence the mixture may be in either of these two proportions; or, taking only half of each of the ingredients, the desired compound will be produced by mixing together 8 lbs. of the highest price tea, 1 lb. of the next highest, 1 lb. of the next, and 5 lbs. of the cheapest; or otherwise, by taking 1 lb. of the highest price, 8 lbs. of the next price, 5 lbs. of the next, and 1 lb. of the cheapest. And generally, in all cases of the kind, the several weights or measures of the component ingredients being determined, as above, they may all be multiplied or divided each by the same number,—any number we please; since the resulting quantities will be to one another still in the same proportion; and therefore however large the quantities of the separate ingredients in either of the sets determined by the foregoing process may be, they may all be reduced, by division, so as to come within any limits, as to quantity, which we may choose to fix. We see, however, that the proportions of the distinct ingredients in the different sets found by the Rule, may themselves be very different:—no common multiplier or divisor applied to the several quantities in the first set above, will give the several quantities of the second set. (See the remarks at page 116.)

2. In what proportion may whiskies at 16*s*., 18*s*., and 22*s*. per gallon be mixed, so that the compound may be worth 20*s*. per gallon?

	16	2 at 16*s*.
	18	2 at 18*s*.
20		
	22	4 + 2 = 6 at 22*s*.

Or, taking the half of each, 1 gallon at 16*s*., 1 gallon at 18*s*., and 3 gallons at 22*s*.: the whole number of gallons being 5, and their value 16*s*. + 18*s*. + 66*s*. = 100*s*., which, divided by 5, gives 20*s*., the price of 1 gallon of the mixture, as it ought to do. And in a similar manner may the accuracy of the general rule, as applied to any particular case, be proved.

3. In what proportion may spirits at 16*s*., 14*s*., 9*s*., and 8*s*., per gallon be compounded in order that the mixture may be worth 10*s*. per gallon? *Ans.* 1 gal. at 16*s*., 2 at 14*s*., 6 at 9*s*., and 4 at 8*s*.

4. In what proportion may raisins at 4*d*., 6*d*., and 10*d*. per lb. be mixed, so that the compound may be worth 8*d*. per lb.?
Ans. 1 lb. at 4*d*., 1 at 6*d*., and 3 at 10*d*.

5. Four sorts of cheap wines at 1*s*. 6*d*., 1*s*. 8*d*., 2*s*., and 2*s*. 4*d*., per quart, respectively, are to be mixed together, so that the compound may be worth 1*s*. 10*d*. per quart: required the quantity of each sort that may be used?
Ans. 1 quart at 1*s*. 6*d*., 3 at 1*s*. 8*d*., 2 at 2*s*., and 1 at 2*s*. 4*d*.
Or 3 at 1*s*. 6*d*., 1 at 1*s*. 8*d*., 1 at 2*s*., and 2 at 2*s*. 4*d*.

[From what is here shown, the reader will readily perceive that the special Rule given in the last article (p. 106) is comprehended in the general Rule above, and that the examples already worked by that Rule may be very conveniently solved by this; we shall here select two of them, and give the solution of each in the margin.

1. Spirits at 12*s*. per gallon are to be mixed with as much water as will reduce the value to 10*s*. per gallon; required the proportion of water to the spirits? (Ex. 2, Prob. 7.) The proportion must be 5 gallons of spirits to 1 gallon of pure water, the cost of the water being 0.

10	12—	10 at 12*s*.,
	0—	and 2 at 0;
		or 5 at 12*s*.,
		and 1 at 0.

2. Spirits at 15*s*. 9*d*. per gallon are to be diluted with sweetened water, at 9*d*. per gallon, in such proportion that the mixture may be worth 13*s*. 3*d*. per gallon; required the proportion? (Ex. 3, Prob. 7.)]

13*s*. 3*d*.	15*s*. 9*d*.—	$12\frac{1}{2}$
	9*d*.—	and $2\frac{1}{2}$,
		or 25 and 5.

PROBLEM 3.

When one of the ingredients is limited to a certain quantity, and the prices only of the other ingredients are given, to find how much of each of these latter may be mixed with the given fixed quantity of the former, in order that the whole mixture may be worth a proposed intermediate price, per pound, gallon, &c.

RULE.—1. Arrange the several prices in column as before; and, as before, take the difference between each and the intermediate price.

2. Then, as the difference written against the price of the given quantity is to any other of the differences, so is the given quantity itself to the quantity against the price of

which that other difference is written. Or, which is the same thing, multiply the given quantity by the difference written against the price of any other of the ingredients, and divide the product by the difference written against the price of the ingredient whose quantity is given: the quotient will be the quantity to be taken of that other ingredient.

Examples.

1. A grocer proposes to mix 20 lbs. of coffee, at 15*d.* per lb., with other coffees at 16*d.*, 18*d.*, and 22*d.* per lb., so that the mixture may be worth 17*d.* per lb.: how much of each of the three latter may he use?

	15	5
	16	1
17		
	18	1
	22	2

Then, per Rule,

$$\left\{\begin{array}{l} 5 : 1 :: 20 : 4 \text{ lbs. at } 16d. \\ 5 : 1 :: 20 : 4 \text{ lbs. at } 18d. \\ 5 : 2 :: 20 : 8 \text{ lbs. at } 22d. \end{array}\right.$$

Or,

20 lbs. $\div$ 5 = 4 lbs., 20 lbs. $\div$ 5 = 4 lbs., 40 lbs. $\div$ 5 = 8 lbs.

Otherwise.

	15	1
	16	5
17		
	18	2
	22	1

Then,

$$\left\{\begin{array}{l} 20 \text{ lbs.} \times 5 = 100 \text{ lbs. at } 16d. \\ 20 \text{ lbs.} \times 2 = 40 \text{ lbs. at } 18d. \\ 20 \text{ lbs.} \times 1 = 20 \text{ lbs. at } 22d. \end{array}\right.$$

[Other suitable mixtures may be obtained by linking the pairs differently.]

Verification of the first set of results.

4	at 16*d.*	=	64*d.*
4	,, 18	=	72
8	,, 22	=	176
and 20	,, 15	=	300
36)			612 (17*d.* per lb.

Verification of the second set of results.

100	at 16*d.*	=	1600*d.*
40	,, 18	=	720
20	,, 22	=	440
and 20	,, 15	=	300
180)			3060 (17*d.*

2. How much wine at 5*s.*, 5*s.* 6*d.*, and 6*s.* per gallon, may be mixed with 3 gallons at 4*s.* per gallon, so that the mixture may be worth 5*s.* 4*d.* per gallon?

	48	8				
	60	$2 \times 3 = 6$, which $\div 8$	$= \frac{3}{4}$	at $60d.$	$= 45d.$
64					
	66	$4 \times 3 = 12$, ,,	$= \frac{3}{2}$	,, 66	$= 99$
	72	$16 \times 3 = 48$, ,,	$= 6$	,, 72	$= 432$
			Also 3	,, 48	$= 144$
			$11\frac{1}{4}$		$720d.$

Multiplying divisor and dividend each by 4 . . 45) 2880 ($64d.$

Here the work is arranged so as to exhibit the required results, and the verification of them, in a more compact form. The answer is $\frac{3}{4}$ gal. at *5s.*, $1\frac{1}{2}$ gallons at *5s. 6d.*, and 6 gallons at *6s.*

Otherwise.

	48	$8 + 2 = 10$,				
	60	$2 + 8 = 10$, which $\times 3$ and $\div 10$	$= 3$	at $60d.$	$= 180d.$
64					
	66	$4 + 16 = 20$, ,,	$= 6$	,, 66	$= 396$
	72	$16 + 4 = 20$, ,,	$= 6$	,, 72	$= 432$
			Also 3	,, 48	$= 144$
			18)		1152(64

Therefore the desired mixture may also be prepared by adding to the 3 gallons at *4s.*, 3 at *5s.*, 6 at *5s. 6d.*, and 6 at *6s.* We would recommend that in these examples the truth of the results be always tested and verified in this way. (See the *General Remarks* at page 116.)

3. A grocer wishes to mix teas at *6s.*, *5s.*, and *3s.* per lb., respectively, with 20 lbs. at *2s.*, so that he may afford to sell the mixture at *4s.*, per lb.: what quantities will suffice for the purpose?
   Ans. Either 20 lbs. at *6s.*, 10 lbs. at *5s.*, and 10 lbs. at *3s.*, or, 20 lbs. at *6s.*, 40 lbs. at *5s.*, and 40 lbs. at *3s.* [See Note below.]

4. Spirits at *7s.* and *4s.* per gallon, respectively, are to be mixed with 40 gallons of other spirits at *12s.* per gallon: what quantities will suffice, in order that the compound may be fairly charged at *8s.* per gallon? *Ans.* 32 gallons of each.

Note.—The prices of the several components in Example 3 are so related, that the question may be answered at once, without employing the pen: we know that *6s.* tea and *2s.* tea, mixed in equal quantities, any whatever, will make *4s.* tea; and so likewise will equal quantities of *5s.* and *3s.* tea. Hence a suitable compound will be obtained, by taking 20 lbs. of the *6s.* tea, and any equal weights whatever of the teas at *5s.* and *3s.*, and then mixing all with the given 20 lbs. at *2s.* The two sets of components above, are those only which the Rule determines. (See the *Remarks* at page 116.)

Problem 4.

When the quantity of each of two or more of the ingredients is given, to find how much of each of the other ingredients will be required to make the mixture of all worth a proposed intermediate price, per pound, gallon, &c.

Rule.—First find the price per lb., gallon, &c., of the mixture formed by combining those ingredients only of which the quantities are given (Prob. 1, p. 108).

Then, regarding this mixture as so many lbs., gallons, &c., of a single ingredient, at a given price per lb., gallon, &c., proceed as in last problem.

Examples.

1. 3 lbs. of tea at 2*s*. per lb., and 6 lbs. at 2*s*. 6*d*., are to be mixed with two other sorts, at 3*s*., and 4*s*., respectively, so that the mixture may be worth 3*s*. 6*d*. per lb., how much of these other sorts must be used for the purpose?

First: 3 lbs. at 2*s*. = 6*s*.
and 6 lbs. ,, $2\frac{1}{2}$*s*. = 15*s*.

9) 21*s*. ($2\frac{1}{3}$*s*. = 2*s*. 4*d*.

We have therefore to mix 9 lbs. at 2*s*. 4*d*. with other teas at 3*s*. and 4*s*., in such quantities as to produce a mixture worth 3*s*. 6*d*. per lb. The proper quantities are to be found by the former Rule, thus:—

	28	6			
	36	6	which × 9 and ÷ 6 =	9 at 36*d*. =	324*d*.
42					
	48	14 + 6 = 20	,,	= 30 ,, 48 =	1440
			Also	9 ,, 28 =	252
				48)	2016 (42*d*.

Hence, if to the given quantities there be added 9 lbs. at 3*s*., and 30 lbs. at 4*s*., making in the whole 48 lbs., the mixture, as here shown, will be worth 42*d*. per lb., as it was required to be.

2. A grocer desires to mix 4 lbs. of coffee, at 1*s*. 6*d*. per lb., and 8 lbs., at 1*s*. 10*d*., with such a quantity of other coffees, at 15*d*. and 16*d*. per lb., respectively, as will make the mixture worth 17*d*. per lb. How much of these other coffees must he use? *Ans.* $14\frac{2}{3}$ lbs. of each.

3. A retailer of spirits mixes 5 gallons of spirits, at 9*s*. 6*d*. per gallon, with 7 gallons, at 10*s*. 6*d*.; and he then desires to add a sufficient quantity, at 13*s*. per gallon, as will make the whole mixture worth 12*s*. per gallon: how much of the latter must he add? *Ans.* 23 gallons.

PROBLEM 5.

When the price per gallon, pound, &c., of each ingredient in the compound, is given, to find the several quantities which may be taken, in order to form a mixture of assigned measure or weight, at an assigned price per gallon, pound, &c.

RULE.—Take the difference between each given price and the assigned intermediate price, as before: then—

As the sum of the differences is to either one of those differences, so is the whole compound to the quantity of that particular ingredient against the price of which the difference thus employed is written.

EXAMPLES.

1. A druggist desires to mix ingredients at 12*d*., 10*d*., 6*d*., and 4*d*. per lb., so as to make a composition of 144 lbs. worth 8*d*. per lb.: what weight of each may he take?

	12	4	48 at 12*d*.	= 576*d*.	
	10	2	24 „ 10	= 240	12 : 4 :: 144 lbs. : 48 lbs.
8					
	6	2	24 „ 6	= 144	12 : 2 :: 144 : 24
	4	4	48 „ 4	= 192	
		12	144)	1152 (8*d*. Proof.	

By linking differently, it will be found that by mixing 24 lbs. at 12*d*., 48 lbs. at 10*d*., 48 lbs. at 6*d*., and 24 lbs. at 4*d*., the desired composition will also be formed. (See the *Remarks* following.)

2. A grocer desires to mix together currants at 11*d*., 9*d*., 6*d*., and 4*d*. per lb., so as to make a mixture of 240 lbs. worth 8*d*. per lb. How many lbs. of each sort may he use?
Ans. 96 lbs. at 11*d*., 48 at 9*d*., 24 at 6*d*., and 72 at 4*d*.

3. Sweet wines at 5*s*., 6*s*., 8*s*., and 9*s*. per gallon, are to be mixed so as to make 87 gallons worth 7*s*. per gallon; how much of each sort will suffice for the purpose?
Ans. 14½ gals. at 5*s*., 29 at 6*s*., 29 at 8*s*., and 14½ at 9*s*.

4. What quantities of the several kinds of coffee, at 15*d*., 17*d*., 18*d*., and 22*d*. per lb., may be mixed together in order to make a composition of 40 lbs. worth 20 pence per lb.?
Ans. 5 lbs. at 15*d*., 5 at 17*d*., 5 at 18*d*., and 25 at 22*d*.

5. Drugs at 8*s*., 5*s*., and 4*s*., per lb. are to be so compounded as to make a mixture of 42 lbs. worth 7*s*. per lb.: required the weight suitable for this purpose of each of the three ingredients?
Ans. 30 lbs. at 8*s*., 6 lbs. at 5*s*., and 6 lbs. at 4*s*.

General Remarks on the Foregoing Problems.

In the foregoing problems and examples, we have comprehended all the varieties of cases which we believe can occur in the actual practice of dealers in compounds. The rules given for determining the proper proportions in which the several distinct ingredients should be mixed together conduct us sometimes to only one set, and sometimes to two or more sets of answers; the number of sets furnished by these rules being always limited. But whenever more than two ingredients, each at a given price, are to be so compounded as to produce a mixture at a proposed intermediate price per unit, whether gallon, lb., &c., the sets of possible proportions are really innumerable; and, by means of algebraic formulæ, we can assign as many of them as we please. We mention this lest the reader should infer, from the language used in the preceding questions, that the answers recorded are the only answers that can be given. We may easily show, without the aid of Algebra, that the answers to some of these questions are innumerable; an instance has indeed been already given in the Note at p. 113:—Take as another instance the worked example at p. 115, in which 144 lbs. at 8*d*. per lb. is to be made up by mixing together four several ingredients at 12*d*., 10*d*., 6*d*., and 4*d*. per lb., respectively. Each of the four equal parts of 144 is 36; and if we write against the highest and the lowest of the given prices any (the same) number less than the number 36, and then against each of the intermediate prices a number as much greater than 36, the four numbers will also express weights of the several component ingredients against the prices of which they are written, suitable for the proposed compound. Thus, writing 30 against the 12*d*. and also against the 4*d*., the intermediate numbers will each be 42; and we shall have

		d.	*d*.				*d*.	*d*.
30	at	12 =	360;	or writing	24	at	12 =	288
42	„	10 =	420	„	48	„	10 =	480
42	„	6 =	252	„	48	„	6 =	288
30	„	4 =	120	„	24	„	4 =	96
144)			1152 (8*d*.		144)			1152 (8*d*.

And the result would have been the same if any other

number below 36 had been chosen, whether whole or fractional; and if this chosen number had been written against the two intermediate prices, instead of against the two extreme prices, the result would have been still the same.

Again: the answer to every such example may be verified in a manner different from that which we have adopted (pp. 109 to 113). In arranging the prices of the several component ingredients in column, we have recommended a small space to be interposed between those of these prices which exceed the proposed mean price, or worth of the compound, and those of them which fall short of this mean price. If the dealer estimate his gain on the component articles on the one side of this interval, and his loss on those on the other side, by selling all of them at the stated intermediate price, the gain and loss must be equal, if the operation be correct; that is, the loss on one side must just balance the gain on the other. Thus taking the example here adduced: the loss on the sale of the 48 at 8*d.* is 48 times 4*d.*, that is, 16*s.*; and the loss on the sale of the 24 at 8*d.* is 4*s.*; the entire loss being £1. But the gain on the sale of the next 24 at 8*d.* is 4*s.*; and the gain on the sale of the following 48 at 8*d.* is 16*s.*; the whole gain being £1, which balances the loss: and similarly in all other like cases.

The reader may turn to the examples, at the pages referred to above, and verify the results in this way. He ought here, however, to be reminded that any verification of the results of an arithmetical operation which has been accurately worked out in accordance with a prescribed Rule is quite superfluous whenever that Rule is previously demonstrated to be true, generally; that is, in every individual case. The Rules in the foregoing article have not been proved to be thus universally true: we have thought it better to give such proof in a supplementary Note, as it is necessary, for the purpose, to use the symbols of Algebra; although, as only the very first principles of that science are brought into operation, we think that no reader of ordinary intelligence will feel any difficulty in following out the reasoning.

Suppose two different ingredients, of given prices per lb. or per gallon, &c., are to be compounded in such proportions that the mixture may fairly bear an assigned inter-

mediate price per lb., per gallon, &c. Let us represent each of these three distinct prices by a letter:—the respective prices (per lb., &c.) of the two ingredients by A and C, and the intermediate price,—the price of the mixture, by B: these three letters will then represent given values, or numbers. Further: let the unknown number of lbs., or gallons, &c., of the commodity at price A, be represented by x: and the unknown number at price C, by y: then, obviously the condition to be satisfied is that A times x + C times y shall be equal to B times x + B times y; that is, there must be the equality (or equation, as it is called)

$$Ax + Cy = Bx + By$$

and therefore, the equality $(A - B)\,x = (B - C)\,y$. And to bring this equality about, it is plain that it is enough that the known quantity $B - C$ should replace the hitherto unknown quantity x, and that the known quantity $A - B$ should replace the hitherto unknown quantity y; for then we should have

$$(A - B)(B - C) = (B - C)(A - B)$$

in which we see that the desired equality is brought about, it being universally true that $A - B$ multiplied by $B - C$ must be equal to $B - C$ multiplied by $A - B$; and this is just what the above form declares.

Hence, arranging as in the margin, the Rule for two ingredients is proved to be true generally; the values of x and y being—

$$B \begin{array}{l} A \text{—} \\ C \text{—} \end{array} \begin{array}{l} x \\ y \end{array}$$

x = the difference between B and C, and y = the difference between A and B. Now from this proof of the Rule, under Prob. 2, for two ingredients, the truth of it for any number of ingredients immediately follows; for the several quantities are always linked together in pairs; and, as here shown, the required condition is always fulfilled for each pair; and therefore, it is necessarily fulfilled for the sum of all the pairs.

In Problem 3, the quantity, as well as the price per lb., per gallon, &c., is also fixed or assigned: thus, in Ex. 2 (p. 112), it is stipulated that there be just "three gallons at 4*s*. per gallon" in the mixture. Were it not for this limitation, the work *would have* been as here annexed; and

		gal.		d.
	48	8	at	48
	60	2	"	60
64				
	66	4	"	66
	72	16	"	72
		30		

there would then have been, as we here see, 30 gallons in the mixture which would, just as correctly, have borne the contemplated price, namely, 5*s.* 4*d.* per gallon. But, instead of 8 gallons at 4*s.*, it is stipulated that there should be only 3; that is, only $\frac{3}{8}$ths of this quantity: consequently, there must be only $\frac{3}{8}$ths of each of the other quantities here determined in the margin, in order that the due proportions may be preserved: and similarly in all such cases. And hence the truth of the Rule.

The reader who satisfies himself in this way of the truth of the two Rules under Problems 2 and 3, cannot have any doubts about the Rules under Problems 4 and 5.

Calculations useful to Goldsmiths, Silversmiths, and Chemists.

[The calculations in the following article are all to be performed by Troy weight.]

Problem 1.

The price of a grain being given, to find the price of an ounce; and, conversely, the price of an ounce being given, to find the price of a grain.

Rule.—The price per grain, in halfpence, will be the price per ounce in pounds (£). And the price, per ounce, in pounds (£), will be the price of a grain in halfpence.

For since there are 24 × 20 grains, that is, 480 grains in an ounce, the price of an ounce, at a halfpenny a grain, is 480 halfpence, or 240 pence; that is £1; so that there must be as many pounds (£) in the price of an ounce, as there are halfpence in the price of a grain. And conversely, there must be as many halfpence in the price of a grain, as there are pounds (£) in the price of an ounce.

Examples.

1. What is the value of an ounce of gold, at the rate of 2*d.* per grain? *Ans.* £4.
2. What is the value of an ounce of gold, at the rate of 1¾*d.* per grain? *Ans.* £3 + £½ = £3 10*s.*
3. The Mint price of gold is £3 17*s.* 10½*d.* per ounce; what is that per grain?

The shillings and pence here are to be expressed in parts of a pound, and decimal parts are the most convenient. A Table of such decimal parts is given towards the end of the book. By referring to this Table, we find that 17*s.* 10½*d.*, expressed in decimals of £1, is £·89375; hence, £3 17*s.* 10½*d.* = £3·89375. Therefore the value of a grain of Mint gold is 3·89375 halfpence. Converting the decimal part of this value into farthings, by multiplying it by 2, the value of the grain is 3 halfpence farthing plus the decimal ·7875 of a farthing, which is $\frac{79}{100}$ths, very nearly; and this again is nearly $\frac{8}{10}$ths or $\frac{4}{5}$ths of a farthing: the answer is therefore 1¾*d.* + $\frac{4}{5}$*f.* nearly.

PROBLEM 2.

The price of a pennyweight being given, to find the price of a pound.

RULE.—As many pence as there are in the price of 1 dwt. (or one-fourth as many farthings), so many pounds (£) will there be in the price of 1 lb. troy.

For there are 240 dwts. in 1 lb. troy, and as many pence in £1, so that each penny in the price of 1 dwt. amounts to £1 in the price of 1 lb.

EXAMPLES.

1. At 4*d.* per dwt., what will 80 lbs. cost? 80 × 4 = £320, *Ans.*
2. At 2¼*d.* per dwt., what will 14 lbs. cost?
   14 × 2¼ = £31½ = £31 10*s.*, *Ans.*
3. At 1¾*d.* per dwt., what will 9 lbs. cost? £1 15*s.* × 9 = £15 15*s.* *Ans.*
4. At ¼*d.* per dwt., what will a lb. cost? *Ans.* 5*s.*
5. If 1 dwt. of silver cost 3¼*d.*, what will 1 lb. cost?
   Ans. £3¼ = £3 5*s.*

PROBLEM 3.

The price of an ounce being given, to find the price of a pound.

RULE.—As many pence as there are in the price of 1 oz., so many shillings will there be in the price of 1 lb. troy. For there are 12 oz. in 1 lb. troy, and as many pence in 1*s.*

EXAMPLES.

1. At 4½*d.* per oz., what is the price of 7 lbs?
   4½ × 7 = 31½*s.* = £1 11*s.* 6*d.*, *Ans.*
2. At 2¾*d.* per oz., what is the price of 11 lbs.?
   2*s.* 9*d.* × 11 = £1 10*s.* 3*d.*, *Ans.*

3. If an ounce of gold cost £3 17*s.* 10½*d.*, what is the price of 1 lb.?
£3 17*s.* 10½*d.* = 934½*d.*; and 934½*s.* = £46 14*s.* 6*d.* *Ans.*

Expressed as the fraction of a pound (£), 14*s.* 6*d.* is $£\frac{14\frac{1}{2}}{20} = £\frac{29}{40}$;

hence, 1 lb. of Mint gold is coined into $46\frac{29}{40}$ sovereigns; or rather (multiplying by 40), 40 lbs. of gold is coined into 1869 sovereigns, as already observed at p. 14.

Problem 4.

The price of an ounce being given, to find the price of any number of pounds, ounces, pennyweights, and grains.

Rule—1. Reduce the pounds to ounces, taking in the ounces given. Consider every ounce as £1, every pennyweight as 1*s.*, and every grain as one halfpenny.

2. Then, whatever part the price per oz. is of £1, that same part of the sum thus obtained will be the answer.

Examples.

1. At 5*s.* 6*d.* per oz., what is the value of 10 lbs. 6 dwts. 14 grs.?

lbs.	dwts.	grs.		oz.	dwts.	grs
10	6	14	=	120	6	14.

Hence, by the Rule:

		£	*s.*	*d.*	
5*s.* = ¼ of a £	4)	120	6	7	= value at £1 per oz.
6*d.* = $\frac{1}{10}$ of 5*s.*	10)	30	1	7¾	
		3	0	$1\frac{39}{40}$	
	Ans.	£33	1*s.*	$9\frac{29}{40}$*d.*	

The reason of the above Rule is this: £1 per ounce troy is one halfpenny per grain; for 24 grains × 20 = number of grains in 1 lb.; and 24 halfpence × 20 = number of halfpence in £1; so that by reckoning the grains as so many halfpence, or half the number as so many pence, the dwts. (24 grains) as so many shillings, and the ounces (20 dwts.) as so many £'s, we express correctly the value of the given weight, at the rate of £1 per oz. If therefore the rate per ounce be only a fractional part of £1, the value of the weight can be only the same fractional part of the value it would have if the rate were a whole £ per ounce.

2. What is the price of a piece of plate, weighing 3 lbs. 7 oz. 14 dwts. 12 grs., at the rate of 7*s.* 6*d.* per oz.?

lbs.	oz.	dwts.	grs.		oz.	dwts.	grs.
3	7	14	12	=	43	14	12.

Hence, by the Rule:

		£	*s.*	*d.*	
5*s.* = ¼ of £1	4)	43	14	6	= value at £1 per oz.
2*s.* 6*d.* = ½ of 5*s.*	2)	10	18	7½	
		5	9	3¾	
		£16	7*s.*	11¼*d.*	= value at 7*s.* 6*d.* per oz.

3. Required the price of 4 lbs. 5 oz. 9 dwts. 10 grs. of plate, at 6*s*. 8*d*. per oz.? *Ans*. £17 16*s*. 5½*d*. + $\frac{2}{3}$*f*.
4. A chased gold vase weighs 1 lb. 3 oz. 4 dwts. 18 grs.; required its value at £5 17*s*. 6*d*. per oz.? *Ans*. £89 10*s*. 4⅞*d*.

In this example, 5 times the value, at £1 per oz., is to be added to the value at 17*s*. 6*d*. per oz. This latter value is most readily computed by subtracting from the value at £1, its ⅛th part; because 17*s*. 6*d*. differs from £1, by the ⅛th part of £1.

Calculations for Articles sold by Avoirdupois Weight.

The following Table for readily ascertaining the price of a hundredweight, from knowing the price of a pound, will often be found useful. It will serve too for the ton as well as for the cwt. (see p. 129).

Table.

For the price of 1 *cwt., at from* ¼*d. to* 3*s*. 6*d. per lb., avoirdupois.*

Price Per lb.	Per cwt.			Price Per lb.	Per cwt.			Price Per lb.	Per cwt.			Price Per lb.	Per cwt.		
d.	£	*s.*	*d.*	*d.*	£	*s.*	*d.*	*d.*	£	*s.*	*d.*	*d.*	£	*s.*	*d.*
¼ is	0	2	4	6 is	2	16	0	11¾ is	5	9	8	20 is	9	6	8
½ „	0	4	8	6¼ „	2	18	4	12 „	5	12	0	21 „	9	16	0
¾ „	0	7	0	6½ „	3	0	8	12¼ „	5	14	4	22 „	10	5	4
1 „	0	9	4	6¾ „	3	3	0	12½ „	5	16	8	23 „	10	14	8
1¼ „	0	11	8	7 „	3	5	4	12¾ „	5	19	0	24 „	11	4	0
1½ „	0	14	0	7¼ „	3	7	8	13 „	6	1	4	25 „	11	13	4
1¾ „	0	16	4	7½ „	3	10	0	13¼ „	6	3	8	26 „	12	2	8
2 „	0	18	8	7¾ „	3	12	4	13½ „	6	6	0	27 „	12	12	0
2¼ „	1	1	0	8 „	3	14	8	13¾ „	6	8	4	28 „	13	1	4
2½ „	1	3	4	8¼ „	3	17	0	14 „	6	10	8	29 „	13	10	8
2¾ „	1	5	8	8½ „	3	19	4	14¼ „	6	13	0	30 „	14	0	0
3 „	1	8	0	8¾ „	4	1	8	14½ „	6	15	4	31 „	14	9	4
3¼ „	1	10	4	9 „	4	4	0	14¾ „	6	17	8	32 „	14	18	8
3½ „	1	12	8	9¼ „	4	6	4	15 „	7	0	0	33 „	15	8	0
3¾ „	1	15	0	9½ „	4	8	8	15½ „	7	4	8	34 „	15	17	4
4 „	1	17	4	9¾ „	4	11	0	16 „	7	9	4	35 „	16	6	8
4¼ „	1	19	8	10 „	4	13	4	16½ „	7	14	0	36 „	16	16	0
4½ „	2	2	0	10¼ „	4	15	8	17 „	7	18	8	37 „	17	5	4
4¾ „	2	4	4	10½ „	4	18	0	17½ „	8	3	4	38 „	17	14	8
5 „	2	6	8	10¾ „	5	0	4	18 „	8	8	0	39 „	18	4	0
5¼ „	2	9	0	11 „	5	2	8	18½ „	8	12	8	40 „	18	13	4
5½ „	2	11	4	11¼ „	5	5	0	19 „	8	17	4	41 „	19	2	8
5¾ „	2	13	8	11½ „	5	7	4	19½ „	9	2	0	42 „	19	12	0

PROBLEM 1.

The price of a dram being given, to find the cost of any number of pounds.

RULE—1. Multiply the number of farthings in the given price of the dram by 16, and the product by the number of lbs.

2. Double the last, or unit's figure of the result for shillings, the remaining figures denoting pounds; and divide the sum, thus expressed, by 6 for the answer.

EXAMPLES.

1. What will 8 lbs. cost, at the rate of $3\frac{1}{4}d.$ a dram?

Here the price of a dram is 13 farthings; hence by the Rule, $13 \times 16 \times 8 = 1664$; and doubling the 4 for shillings, we have £166 8*s*. $\div$ 6 = £27 14*s*. 8*d*. *Ans.*

This short and convenient Rule rests mainly on the following principle, namely: If any number of pounds (£) be divided by a number terminating in 0, the quotient will be the same as if we replace the final figure in the dividend by double that number of shillings, and then suppress the 0 in the divisor; as for instance—

$$\frac{£734}{30} = \frac{£73\ \ 8s.}{3};\ \frac{£734}{70} = \frac{£73\ \ 8s.}{7};\ \frac{£2587}{60} = \frac{£258\ \ 14s.}{6};$$

and so on; for the second form of expression, in every such case, is restored to the first form by multiplying numerator and denominator by 10. The final figure in the dividend, which is so many pounds, being regarded as shillings, and doubled, and then multiplied by 10, is thus taken 20 times; so that the shillings are reconverted into the pounds which they replaced.

This being understood, and knowing that the price of a lb., in farthings, is $16 \times 16 \times$ price of a dram in farthings, we must divide this product by $4 \times 12 \times 20$ to bring the price of 1 lb. into pounds (£). But

$$\frac{\text{Price of dr.} \times 16 \times 16}{4 \times 12 \times 20} = \frac{\text{Price of dr.} \times 16^{*}}{60}$$

* This more simple form of the fraction is not the result of performing the multiplications indicated in the first form, and then reducing

This expresses the number of pounds (£) in the price of a lb., the factor "Price of dr.," standing for the number of farthings in the price of a dram. The unabbreviated Rule would therefore be to multiply this number of farthings by 16, and to divide the product by 60, in order to get the price, in pounds (£), of a single lb.; but by the foregoing principle, the Rule becomes abridged to that given above.

2. At $4\frac{1}{2}d.$ the dram, what will 68 lbs. cost? *Ans.* £326 8*s.*
3. At $\frac{1}{4}d.$ the dram, what will 1 lb. cost? *Ans.* 5*s.* 4*d.*
4. Required the cost of 7 lbs., at $3\frac{3}{4}d.$ per dram? *Ans.* £28.

NOTE.—It is worth while to observe here, that all examples coming under the present problem, may also be readily worked by help of the fact, that at $\frac{1}{4}d.$ the dram, the cost of 1 lb. is 5*s.* 4*d.* Thus taking Ex. 1, the cost of 1 lb., at 13 farthings per dram, is 5*s.* 4*d.*$\times$13=£3 9*s.* 4*d.*; and therefore the cost of 8 lbs. is £27 14*s.* 8*d.*

PROBLEM 2. (CONVERSE OF PROB. 1.)

The price of 1 *lb. being given, to find the price of a dram.*

RULE.—Divide the number of pence in the price of 1 lb. by 64; the quotient will be the number of farthings in the price of a dram. The reason is obvious from the NOTE above, seeing that 5*s.* 4*d.* is 64 pence.

EXAMPLES.

1. If 1 lb. cost £3 9*s.* 4*d.*, what will a dram cost?
   69*s.* 4*d.* = 832*d.*; and 832 ÷ 64 = 13*f.* = $3\frac{1}{4}d.$
2. What is the price of a dram when 1 lb. costs £4? *Ans.* $3\frac{3}{4}d.$
3. If 1 lb. cost £2 15*s.* 8*d.*, what will a dram cost? *Ans.* $2\frac{1}{2}d.$, $\frac{7}{16}f.$

PROBLEM 3.

The price of an ounce being given, to find the price of 1 *lb.*

RULE.—Regard the given price, in farthings, as so many shillings, and divide these shillings by 3.

the fraction to one of lower terms. This kind of work is altogether dispensed with, and the simplified form written down at once, by merely eliminating the factors common to numerator and denominator in the first form. We know that 16 in the numerator is 4 $\times$ 4, and that 4 $\times$ 12 in the denominator is 4 $\times$ 4 $\times$ 3 (the 12 being 4 $\times$ 3); dismissing then these 4's, there remain but 16 in the numerator, and *3 $\times$ 20 or 60* in the denominator.

For taking the farthings for shillings, is the same as multiplying by 48, or by 16 and 3: dividing then the product by 3, we get the price of an ounce, multiplied by 16; that is, the price of 16 oz., or 1 lb. avoirdupois.

Examples.

1. What will 1 lb. avoirdupois come to, at $7\frac{1}{2}d.$ per oz.?
   $7\frac{1}{2}d. = 30$ farthings; therefore the price of 1 lb. is $30s. \div 3 = 10s.$
2. What will 1 lb. cost, at $10\frac{3}{4}d.$ per oz.? *Ans.* $14s.\ 4d.$
3. What will 7 lb. come to, at $2\frac{1}{4}d.$ per oz.? *Ans.* £1 $1s.\ 0d.$
4. What will 13 lb. come to, at $3\frac{3}{4}d.$ per oz.? *Ans.* £3 $5s.\ 0d.$

Problem 4. (Converse of Prob. 3.)

The price of 1 *lb. being given, to find the price per ounce.*

Rule.—Regard the given price, in shillings, as so many farthings, and multiply these farthings by 3.

The truth of this Rule is plain from that of the former Rule.

Examples.

1. At $6s.$ per lb., what will an ounce cost? $6f. \times 3 = 18f. = 4\frac{1}{2}d.$ *Ans.*
2. At $9s.$ per lb., the price of 1 oz. is $6\frac{3}{4}d.$ *Ans.*
3. At $4s.\ 9d.$ per lb., the price of 1 oz. is $3\frac{1}{2}d. + \frac{1}{4}f.$ *Ans.*

For $4s.\ 9d. = 4\frac{3}{4}s.$; and this number of farthings, multiplied by 3, gives for product $12\frac{9}{4}f. = 14\frac{1}{4}f. = 3\frac{1}{2}d. + \frac{1}{4}f.$

Problem 5.

The price of 1 *oz. being given, to find the price of* 1 *stone, of* 1 *qr., and of* 1 *cwt.*

Rule I.—Find the price of 1 lb., by Prob. 3, and thence the price of 14 lb. (or a stone). Multiply this by 2, for the price of a quarter, and by 8, for the price of a cwt. The reason is obvious.

Rule II.—Multiply the pence per oz. by the number of cwts.; and whatever part of £1 the product is, take that part of £1792: the result will be the price of 1 cwt. in £ *s. d.*

The number 1792 is the product of 112 by 16; so that £1792 is the value of 1 cwt., at £1 per oz.; and therefore, whatever fraction of £1 the price of 1 oz. may be, the same fraction of £1792 must be the price of 1 cwt.; or whatever fraction of £1 the price of any number of ounces may be, the same fraction of £1792 must be the price of as many cwts.

correct to the nearest farthing. And so in other cases; and if the price per quarter, or per stone, were given, we should enter the Table with 4 times, or 8 times this price, against which would be found the price per lb. to the nearest farthing, if not exactly.

Problem 6.

The price of 1 lb. being given, to find the price of a ton.

Rule I.—Find the price of 1 cwt. by Rule III., p. 126; regard the shillings in this price as so many pounds, and the pence as fractional parts; and the answer in pounds will be expressed. Or—

Rule II.—Multiply the number of farthings in the price of the lb. by 7, and divide the product by 3, for the answer in pounds.

The truth of the first of these two Rules will be seen by considering that by taking the shillings in the price of 1 cwt. as so many pounds, we must get the price of 20 cwt. And the truth of the second Rule follows from the fact that by replacing the price in farthings by so many pounds, we virtually multiply that price by $4 \times 12 \times 20$; therefore multiplying the number of farthings (taken as pounds) by 7, and then dividing by 3, the price per lb. is multiplied by $4 \times 12 \times 20 \times 7 \div 3 = 4 \times 4 \times 20 \times 7 = 28 \times 4 \times 20$; and 28 lbs. = 1 qr., 4 qrs. = 1 cwt., and 20 cwt. = 1 ton.

We thus see that by the contrivance of multiplying by 7 and dividing by 3, the price per lb. is virtually multiplied by 28, 4, and 20; thus giving for product the price of a ton.

Examples.

1. If 1 lb. cost 7*d.*, what will a ton cost?

By Rule I.

9*s.* 4*d.* × 7 = 65*s.* 4*d.* = price of 1 cwt.
£65⅓ = £65 6*s.* 8*d.* = price of 1 ton.

By Rule II.

£28 × 7 ÷ 3 = £196 ÷ 3 = £65 6*s.* 8*d.* *Ans.*

2. If 1 lb. cost 1¾*d.*, what will a ton cost?

By Rule II. £7 × 7 ÷ 3 = £49 ÷ 3 = £16 6*s.* 8*d.* *Ans.*

3. If 1 lb. cost 10*d*., what will a ton cost?

By Rule I.

9*s*. 4*d*. $\times$ 10 $=$ 93*s*. 4*d*.
£93⅓ $=$ £93 6*s*. 8*d*. *Ans*.

By Rule II.

£40 $\times$ 7 $\div$ 3 $=$ £280 $\div$ 3 $=$ £93 6*s*. 8*d*. *Ans*.

As by Rule II. the divisor is 3, the fraction of £1, which gives the shillings and pence, must always be either £⅓, or £⅔; that is, either 6*s*. 8*d*. or 13*s*. 4*d*.

4. If 1 lb. cost 4¾*d*., what will a ton cost?

By Rule II., £19 $\times$ 7 $\div$ 3 $=$ £133 $\div$ 3 $=$ £44 6*s*. 8*d*. *Ans*.

5. If 1 lb. cost 2¾*d*., what will a ton cost? *Ans*. £25 13*s*. 4*d*.
6. If 1 lb. cost 4¼*d*., what is a ton worth? *Ans*. £39 13*s*. 4*d*.

Note.—The Table at page 122, for finding, by inspection, the price of 1 cwt., from that of 1 lb. being given, may be advantageously employed for also finding the price of a ton; for a ton being 20 cwt., we have only to regard every shilling, in the price there, of 1 cwt., as £1, and every 4*d*. as 6*s*. 8*d*. Thus, taking Example 4, above, and referring to the Table, we find against 4¾*d*., 44*s*. 4*d*., which, for a ton, we should read as £44 6*s*. 8*d*. Again, taking Ex. 5, the Table gives 25*s*. 8*d*., to be read as £25 13*s*. 4*d*.; and similarly in other cases.

Problem 7. (Converse of Prob. 6.)

The price of a ton being given, to find the price of 1 *lb.*

Rule.—Multiply the price of the ton, expressed in £'s and fractions of a £, by 3, and then divide by 7: the quotient will be the number of farthings in the price of 1 lb.

Examples.

1. If a ton of iron cost £16 6*s*. 8*d*., what is the price per lb.?
   £16 6*s*. 8*d*. $=$ £16⅓, and 16⅓ $\times$ 3 $\div$ 7 $=$ 49 $\div$ 7 $=$ 7 farthings.
2. If a ton cost £39 13*s*. 4*d*., what is the price per lb?
   £39 13*s*. 4*d*. $=$ £39⅔, and 39⅔ $\times$ 3 $\div$ 7 $=$ 119 $\div$ 7 $=$ 17 *f*. $=$ 4¼*d*.
3. If a ton cost £84, what will 1 lb. cost?
   84 $\div$ 7 $=$ 12, and 12 $\times$ 3 $=$ 36 *f*. $=$ 9*d*.
4. If a ton cost £36 10*s*., what will 1 lb. cost?
   £36 10*s*. $=$ £36½, and £36½ $\times$ 3 $\div$ 7 $=$ 109½ $\div$ 7 $=$ 15$\frac{9}{14}$ farthings; that is, 3¾*d*. and the $\frac{9}{14}$th of a farthing.

It will be observed that in each of the examples which

precede this last, the given price is such that when it is multiplied by 3, the product is an integral number of pounds, —without shillings or pence. In this example the product involves a fraction, namely, $\frac{1}{2}$. Whenever such is the case, the answer to the question will always be a certain number of farthings and some fraction of a farthing besides. 1 lb. therefore could not be purchased at its exact value in existing coin; consequently, in order that the seller of a single lb. may not sustain loss, the fraction must be rejected, and an additional farthing added. In the case before us, less than a stone, or 14 lbs., could not be sold at its exact value; but since $15\frac{9}{14} \times 14 = 219$, the exact price of a stone would be 219 farthings, or 4s. $6\frac{3}{4}d$. And even though 3 times the price of the ton be an integral number of £'s, yet a fraction of a farthing must form part of the exact price of 1 lb., if the product by the 3 be not exactly divisible by 7.

5. If a ton cost £42, what is the price per lb? *Ans.* $4\frac{1}{2}d$.
6. If a ton cost £23 6s. 8d., how much is that per lb? *Ans.* $2\frac{1}{2}d$.
7. If a ton cost £43 15s., what will 1 lb. cost? *Ans.* $4\frac{1}{2}d. + \frac{3}{4}f$.

In order to prove the truth of the foregoing Rule, we need only observe that the price of the ton being expressed in £'s, if we multiply the price by $20 \times 12 \times 4$, we shall get the price in farthings; and from this, to get the price of a lb., we must divide it by 20×112; or, which is the same thing, we must divide 12×4 times the price in £'s by 112, the common factor 20 being suppressed in both multiplier and divisor. (See foot-note, p. 123.) But

$$\frac{12 \times 4}{112} = \frac{3 \times 16}{7 \times 16} = \frac{3}{7};$$

the factor 16 in both numerator and denominator being suppressed; so that we have only to multiply the price of the ton, in £'s, by 3, and then divide by 7; which is the Rule.

Problem 8.

The price of 1 *cwt. being given, to find the price of any number of tons.*

Rule.—Multiply the number of pence in the price of the *cwt. by the* number of tons, and divide the product by 12:

the quotient will be the number of £'s in the price of the tons. [Or 20 times the price of 1 cwt. is the price of a ton.]

For by multiplying the given price, in pence, by 20, we get the price, in pence, of a ton; and by dividing this number of pence by 12 and 20, we get the number of pounds in the price of a ton, and thence the price, in pounds, of any number of tons. But 20 being here both a multiplier and a divisor may be expunged, and hence the Rule.

EXAMPLES.

1. 6 tons, at 2*d.* per cwt. $= 2 \times 6 \div 12 = 1$: therefore £1 is the price of the 6 tons. Or 2*d.* per cwt. is 40*d.* or 3*s.* 4*d.* per ton; therefore the price of 6 tons is £1.
2. 8 tons, at 3*d.* per cwt. $= 3 \times 8 \div 12 = 2$; therefore the price is £2.
3. 24 tons, at $7\frac{1}{2}$*d.* per cwt. $= 7\frac{1}{2} \times 24 \div 12 =$ £15
4. 36 tons, at $13\frac{1}{2}$*d.* per cwt., is $13\frac{1}{2} \times 36 \div 12 =$ £$40\frac{1}{2}$ = £40 10*s.*
5. Required the price of 60 tons, at $19\frac{1}{2}$*d.* per cwt.? *Ans.* £97 10*s.*

NOTE.—If either of the two numbers to be multiplied together be divisible by 12, the division should be executed before the multiplication: thus, in Examples 3, 4, and 5, the use of the pen is scarcely necessary, seeing that 24, 36, and 60, by division by 12, give the small multipliers 2, 3, and 5; and that twice $7\frac{1}{2}$, 3 times $13\frac{1}{2}$, and 5 times $19\frac{1}{2}$, give products which no pen-work is needed to determine.

PROBLEM 9. (CONVERSE OF PROB. 8.)

The price of a ton being given, to find the price of 1 *cwt.*

RULE.—Express the price of the ton in pounds (£) and fractions of a £; then the price of a cwt. will be that number of shillings. This is obvious, because the price of 1 cwt. must be the twentieth part of the price of a ton, and the twentieth part of any number of pounds is that same number of shillings.

EXAMPLES.

1. If a ton cost 3*s.* 4*d.*, what will 1 cwt. cost?
   3*s.* 4*d.* = £$\frac{1}{6}$; and $\frac{1}{6}$*s.* = 2*d.* *Ans.*
2. If a ton cost 14*s.* 2*d.*, what will 1 cwt. cost?

What fraction of one pound 14*s.* 2*d.* is, is not readily seen: we know however that 6*s.* 8*d.* = £$\frac{1}{3}$; and that 6*s.* 8*d.* + 7*s.* 6*d.* = 14*s.* 2*d.*; also that 7*s.* 6*d.* = £$\frac{3}{8}$. Consequently, 14*s.* 2*d.* = £$\frac{1}{3}$ + £$\frac{3}{8}$: hence the price of 1 cwt. is $\frac{1}{3}$*s.* + $\frac{3}{8}$*s.* = 4*d.* + $4\frac{1}{2}$*d.* = $8\frac{1}{2}$*d.* *Ans.*

3. If a ton cost 12*s.* 8*d.*, what will 1 cwt. cost?

> 12*s.* 8*d.* = 10*s.* + 2*s.* 6*d.* + 2*d.* = £$\frac{1}{2}$ + £$\frac{1}{8}$ + 2*d.*, therefore,
> $\frac{1}{2}$*s.* + $\frac{1}{8}$*s.* + $\frac{2}{20}$*d.* = 6*d.* + 1$\frac{1}{2}$*d.* + $\frac{1}{10}$*d.* = 7$\frac{1}{2}$*d.* + $\frac{2}{5}$ farthing. *Ans.*

We here divide the odd twopence by 20, because the 20th part of the entire sum, in the price of 20 cwt., must be taken to get the price of 1 cwt. And in imitation of this manner of computing must every case be treated in which overplus pence remain after the fractions of £1 are subducted from the given price of the ton.

4. If a ton cost 12*s.* 6*d.*, what will 1 cwt. cost? *Ans.* 7$\frac{1}{2}$*d.*
5. If a ton cost £1 12*s.* 6*d.*, what will 1 cwt. cost? *Ans.* 1*s.* 7$\frac{1}{2}$*d.*
6. If a ton cost £1 2*s.* 6*d.*, what will 1 cwt. cost? *Ans.* 1*s.* 1$\frac{1}{2}$*d.*
7. If a ton cost 17*s.* 10*d.*, what is the price of 1 cwt.? *Ans.* 10$\frac{1}{2}$*d.* + $\frac{4}{5}$*f.*

Problem 10.

The price of 1 cwt. being given, to calculate the cost of any number of cwts., qrs., and lbs.

Rule.—Write down first as many pounds (£) as there are cwts.; then, for every quarter, put 5*s.*, and for every lb., write 2$\frac{1}{7}$*d.* We shall thus have the value of the proposed weight at £1 per cwt. Take parts for the shillings and pence, in the given price per cwt., and the answer will be obtained. [If the price per cwt. be less than £1, see Rule p. 90.]

For at £1 per cwt., the value of 1 qr. is the fourth part of £1, that is, 5*s.*; and the value of 1 lb. is the 28th part of this, namely 60*d.* ÷ 28 = 2$\frac{1}{7}$*d.*; so that whatever fractional parts of £1 the actual price of the cwt. may be or may include, the same parts of these several amounts must be taken to obtain the correct answer to the question.

Examples.

1. What is the price of 13 cwts. 2 qrs. 14 lbs., at £1 6*s.* 8*d.* per cwt.?

	£	*s.*	*d.*
The weight is 13 cwts. 2$\frac{1}{2}$ qrs., and the price at £1 is	13	12	6
and at 6*s.* 8*d.*, or £$\frac{1}{3}$ it is	4	10	10
Hence the whole price is	£18	3	4*d.* *Ans.*

2. What will 19 cwts. 3 qrs. 19 lbs. cost, at £4 15*s*. 6*d*. per cwt.?

	£	s.	d.	
	19	15	0	
4*s*. 0*d*. = £$\frac{1}{5}$		3	4$\frac{5}{7}$	= 2$\frac{1}{7}$*d*. × 19
4*s*. 0*d*. = £$\frac{1}{5}$				
5*s*. 0*d*. = £$\frac{1}{4}$	19	18	4$\frac{5}{7}$	= Price at £1
2*s*. 6*d*. = £$\frac{1}{8}$ = $\frac{1}{2}$ of £$\frac{1}{4}$			4	
15*s*. 6*d*.	79	13	6$\frac{6}{7}$	= Price at £4
	3	19	8$\frac{1}{7}$	= $\frac{1}{5}$th price at £1
	3	19	8$\frac{1}{7}$	= ditto
	4	19	7$\frac{5}{28}$	= $\frac{1}{4}$th „
	2	9	9$\frac{23}{56}$	= $\frac{1}{8}$th „

Ans. £95 2*s*. 3$\frac{41}{56}$*d*. = £95 2*s*. 4*d*. *nearly*.

There are, of course, various ways of cutting up the shillings and pence into fractions of £1: the computer must be left to his own judgment and sagacity as to the choice of the most convenient subdivision. The subdivision here might have been 10*s*. + 5*s*. + 6*d*. = 15*s*. 6*d*.

3. What is the price of 27 cwt. 2 qrs., at £2 14*s*. 6*d*. per cwt.?
   Ans. £74 18*s*. 9*d*.
4. What is the price of 42 cwt. 1 qr. 20 lbs., at £1 13*s*. 4*d*. per cwt.?
   Ans. £70 14*s*. 3$\frac{3}{7}$*d*.
5. What will 85 cwt. 1 qr. 10 lbs. come to, at £2 17*s*. 6*d*. per cwt.?
   Ans. £245 7*s*. 0$\frac{3}{28}$*d*.

It is scarcely necessary to say that the fractions of a penny, retained in these answers, would be disregarded in actual practice: they are given here, as in several other of the results in this book, chiefly that the reader may put to the test his ability to compute with fractions.

Calculations respecting Land, Timber, &c.

Problem 1.

The price of a perch being given, to find the price of an acre.

Rule.—Regard the price of the perch, in pence, as so many pounds, and from that number of pounds subtract a third part; the remainder will be the price of an acre.

For, by regarding the price, in pence, as so many pounds, we, in effect, multiply that price by 20 and by 12; that is, by

20×12 or 240. Now the number of perches in an acre is 40×4 (Table, p. 23), so that if the price of a perch be multiplied by 40×4 (that is, by 160), the result will be the price of an acre. But two-thirds of 20×12 are 40×4 (twice the 20 multiplied by a third of the 12); consequently two-thirds of the price, in pence, of a perch, regarding those pence as so many pounds, must be the price of an acre, and two-thirds of anything is the whole minus one-third; hence the Rule.

Examples.

1. What is the price of an acre, at the rate of 15¾*d.* per perch? £15 15*s.* ÷ 3 = £5 5*s.*, therefore the price is £10 10*s.* per acre.
2. What is the price of an acre, at the rate of 2*s.* 7½*d.* per perch? £31 10*s.* ÷ 3 = £10 10*s.*, therefore the price is £21. *Ans.*
3. What is the price of an acre, at 1*s.* 8¼*d.* per perch? *Ans.* £13 10*s.*
4. What is the price of an acre, at 3*s.* 10¾*d.* per perch? *Ans.* £31 3*s.* 4*d.*

Problem 2.

The price of a perch being given, to find the price of a rood.

Rule.—One-sixth as many pounds as the perch costs pence will be the price of a rood.

For we have seen (last Prob.) that $\frac{2}{3}$rds of this number of pounds is the price of an acre; so that, dividing the $\frac{2}{3}$rds by 4, we must get the price of a rood. But $\frac{2}{3} \div 4 = \frac{2}{12}$, and $\frac{2}{12} = \frac{1}{6}$; hence the Rule.

Examples.

1. If a perch cost 6*d.*, what will a rood cost? £6 ÷ 6 = £1, *Ans.*
2. If a perch cost 2*s.* 7½*d.*, what will a rood cost? £31 10*s.* ÷ 6 = £5 5*s.*, *Ans.*

Note.—This problem may obviously be made to supersede the former one, for we have only to multiply the price of a rood by 4 to get the price of an acre; thus, 4 times the sum which is the answer to this last question is the answer to question 2 of last problem.

3. What is the price of a rood, if a perch cost 15¾*d.*? *Ans.* £2 12*s.* 6*d.*
4. What is the price of a rood, if a perch cost 1*s.* 8¼*d.*? *Ans.* £3 7*s.* 6*d.*

Problem 3. (Converse of Probs. 1 and 2.)

The price of an acre, or of a rood, being given, to find the price of a perch.

Rule.—*For the acre.* To the price of the acre, in pounds, add its half; the result will be the number of pence in the price of a perch.

For the rood. Six times the number of pounds in the price of a rood will be the number of pence in the price of a perch.

Examples.

1. At £10 10*s.* per acre, what is the price of a perch?
   £10 10*s.* × £5 5*s.* = £15 15*s.*; therefore, 15¾*d.* = price of a perch.

It has already been shown (Prob. 1) that two-thirds of the number of pence in the price of a perch are the number of pounds in the price of an acre; consequently the whole number of pence in the price of a perch must be equal to the number of pounds in the price of an acre, and half that number of pounds besides; because the whole of anything is ⅔rds of it and half as much more, and hence the truth of the Rule.

2. If a rood of land cost £2 12*s.* 6*d.*, what will a perch cost?
   £2 12*s.* 6*d.* = £2½ + £⅛; and 6 times this is £15 + £¾.
   Hence, by the Rule, 15¾*d.* is the price of a perch.

If it had been an acre, instead of a rood, we should have multiplied the number of pounds in the price by 1½; but as it is, we ought to multiply by 4 times this, since the perch is 4 times as dear as it would be if the assigned price had been per acre instead of per rood. Hence we must multiply by 1½ × 4 = 6, as the Rule directs.

3. At £13 10*s.* per acre, what is the price of a perch? *Ans.* 1*s.* 8¼*d.*
4. At £31 3*s.* 4*d.* per acre, what is the price of a perch?
   Ans. 3*s.* 10¾*d.*
5. If a rood cost £5 5*s*, what is the price of a perch? *Ans.* 2*s.* 7½*d.*
6. If a rood cost £3 7*s.* 6*d.*, what will a perch cost? *Ans.* 1*s.* 8¼*d.*

Problem 4.

The price of a square yard of land being given, to find the price of an acre.

Rule I.—Multiply the number of pence, in the price per yard, by 20, and consider the result as so many pounds, to

which add as many times 3*s*. 4*d*. as there are pence in the given price; the sum will be the price per acre.

The price of an acre may, of course, be found by multiplying the price of a square yard by the number 4840, because there are 4840 square yards in an acre; or, which is the same thing, by multiplying 4840 pence by the number of pence in the price of a square yard. Now by regarding these pence as so many pounds, we virtually multiply them by 240; and by again multiplying by 20, we multiply, on the whole, by 4800, which is 40 short of the number 4840; hence, to the product thus obtained, we must add 40 times the number of pence in the price of the yard in order to get the full price of an acre; or, which is the same thing, we must add 40 pence multiplied by the number of pence in the price of the yard. But 40 pence is 3*s*. 4*d*.; and hence the Rule.

Examples.

1. If a square yard be worth 2*d*., what is the value of an acre?
   £2 $\times$ 20 = £40; which, with twice 3*s*. 4*d*., gives £40 6*s*. 8*d*. *Ans.*
2. If a square yard be worth $3\frac{1}{4}$*d*., what is an acre worth?
   £$3\frac{1}{4}$ $\times$ 20 = £65; and $3\frac{1}{4}$ times 3*s*. 4*d*. is 10*s*. 10*d*.: therefore £65 10*s*. 10*d*. is the value of an acre.

Note.—It is easy to see that the Rule may be varied a little in its expression. It may be as follows: £20 3*s*. 4*d*. being 4840*d*., and £5 0*s*. 10*d*. = 4840*f*.

Rule II.—Multiply £20 3*s*. 4*d*., by the number of pence in the price of a square yard: the product will be the price of an acre. Or, multiply £5 0*s*. 10*d*. by the number of farthings in the price of the square yard; since a farthing per sq. yard is £5 0*s*. 10*d*. per acre. Or, lastly: take the number of pence in the price of a sq. yd. as so many pounds (£), and multiply that number of pounds by $20\frac{1}{6}$. Thus, in Ex. 2, above, the operation by this last Rule will be that here annexed; remembering that £$3\frac{1}{4}$ = £3 5*s*.

£	s.	d.
3	5	0
		20
65	0	0
	10	10
£65	10*s*.	10*d*.

3. If a square yard cost $1\frac{3}{4}$*d*., what will an acre cost? *Ans.* £35 5*s*. 10*d*.
4. If a square yard cost $2\frac{1}{4}$*d*., what will an acre cost? *Ans.* £45 7*s*. 6*d*.
5. What is the price of an acre, at the rate of $2\frac{1}{2}$*d*. per square yard?
   Ans. £50 8*s*. 4*d*.

PROBLEM 5. (CONVERSE OF PROB. 4.)

The price of an acre being given, to find the price of a square yard.

RULE.—Divide the price of the acre, expressed in pounds and fractions of a pound, by £$20\frac{1}{6}$: the quotient will be the number of pence in the price of a square yard.

For in the preceding operations, in order to get the price of an acre from that of a yard, we multiply £20 by the number of pence in the price of the yard, and also 3*s.* 4*d.* by that number of pence; that is to say, we multiplied £20 3*s.* 4*d.* by the number of pence in the yard, in order to get the price per acre. Hence, in the reverse operation, we must divide the price of the acre, in pounds, by £$20\frac{1}{6}$, this being the equivalent of £20 3*s.* 4*d.*

EXAMPLES.

1. If an acre of land be worth £65 10*s.* 10*d.*, what is the worth of a square yard?

   £65 10*s.* 10*d.* (see Table, p. 50) $=$ £$65\frac{1}{2}$ $+$ £$\frac{1}{24}$, or £$\frac{131}{2}$ $+$ £$\frac{1}{24}$.
   Dividing each by $20\frac{1}{6}$, that is, multiplying by $\frac{6}{121}$, the fractions are $\frac{393}{121}+\frac{1}{484}=\frac{1572+1}{484}=\frac{1573}{484}=3\frac{1}{4}d.$

NOTE.—The converse of a problem is usually more troublesome to work than the direct problem itself. It is likely that, in the present converse problem, some might find it easier to proceed according to the Rule of common arithmetic; namely, to reduce the £65 10*s.* 10*d.* to pence, and then to divide by 4840.

2. If an acre be worth £45 7*s.* 6*d.*, what is a square yard worth?
   Ans. $2\frac{1}{4}d.$
3. If an acre be worth £50 8*s.* 4*d.*, what is a square yard worth?
   Ans. $2\frac{1}{2}d.$

PROBLEM 6.

To reduce Irish acres to English, and English to Irish.

RULE.—Multiply the number of Irish acres by 196, and divide by 121: the result will be the number of English acres.

Multiply the number of English acres by 121, and divide by 196: the result will be the number of Irish acres. For an Irish acre contains 7840 yards, and an English acre 4840; and these numbers are in the ratio of 196 to 121.

Examples.

1. How many English acres are there in 484 Irish acres?
   $484 \times 196 = 94864$; and this $\div$ 121 (or by 11 and 11) gives 784; or more simply, $4 \times 196 = 784$, the *Ans.*
2. How many Irish acres are there in 784 English acres?
   $784 \times 121 \div 196 = 484$; or $4 \times 121 = 484$ Irish acres, the *Ans.*
3. How many English acres are there in $134\frac{1}{2}$ Irish acres?
   Ans. $217\frac{7}{8}$ very nearly.
4. How many Irish acres are there in $273\frac{1}{4}$ English acres?
   Ans. $168\frac{9}{13}$ very nearly.

Problem 7.

To reduce Irish miles to English, and English to Irish.

The English mile contains 1760 yards, and the Irish mile 2240 yards, which two numbers are to one another as 11 is to 14, or as 1 to $1\frac{3}{11}$; so that an Irish mile is $1\frac{3}{11}$ times as long as an English mile; or 11 Irish miles make 14 English. Hence this

Rule.—Increase the number of Irish miles by the $\frac{3}{11}$ part of that number: the result will be the number of English miles.

Diminish the number of English miles by the $\frac{3}{14}$ part of that number; the result will be the number of Irish miles. Or, which is the same thing, multiply the number of Irish miles by 14, and divide the product by 11; the quotient will be the number of English miles.

Multiply the number of English miles by 11, and divide by 14 (or by 7 and 2); the quotient will be the number of Irish miles.

Note.—When the number of English miles is even, we may take half that number only, using for divisor 7 instead of 14, as in Ex. 2.

Examples.

1. How many English miles are there in 27 Irish miles?
   $27 + \frac{81}{11} = 27 + 7\frac{4}{11} = 34\frac{4}{11}$, the number of English miles.
   Or thus; $27 \times 14 = 378$; and this $\div$ 11 gives $34\frac{4}{11}$.
2. How many Irish miles are there in 40 English miles?
   $40 - \frac{120}{14} = 40 - 8\frac{4}{7} = 31\frac{3}{7}$, the number of Irish miles.
   Or thus (see preceding Note above), $20 \times 11 = 220$, which $\div$ 7 gives $31\frac{3}{7}$.
3. How many English miles are there in 75 Irish miles? *Ans.* $95\frac{5}{11}$.
4. How many Irish miles are there in 95 English miles? *Ans.* $74\frac{9}{14}$.

The foregoing calculations, respecting Land, must suffice for the present small volume. It would, of course, be out of place here to enter upon the general subject of Surveying. We shall now therefore proceed to some useful particulars connected with the measurement of TIMBER.

TIMBER TRADE.

I. Superficial Measure.

When timber is sold in planks,—or in logs for the purpose of being cut into planks,—its price is estimated by superficial measurement; that is, by the number of square feet contained under the length and breadth of the plank; the price per square foot differing, of course, for different thicknesses of plank.

PROBLEM 1.

To find the number of square feet in a board or plank.

RULE.—Multiply the length by the breadth; the product will be the number of square feet, when the plank is of the usual form; that is, a rectangle.

But if the plank be tapering, add the two widths at the ends together, and take half the sum for the mean breadth, by which multiply the length: the result will be the number of square feet.

[We have here spoken of multiplying length by breadth, as it is the universal form of expression in the trade: but, in fact, only abstract numbers, and not the concrete quantities, feet and inches, are really worked with. See the remarks on this subject at p. 37.]

EXAMPLES.

1. How many square feet are there in a plank 16 feet 6 inches long, and 14 inches broad?

ft.	in.	
16	6	or: 16 ft. 6 in. = 198 in.; and 198 × 14 = 2772.
1	2	144) 2772 ($19\frac{36}{144}$ = $19\frac{1}{4}$ sq. ft. *Ans.*
16	6	
2	9	
19 Ft.	3 Pts.	*Ans.* $19\frac{3}{12}$ sq. ft. = $19\frac{1}{4}$ sq. ft.

be overlooked, that whatever be the proposed thickness of the planks, unless the breadth of the log contain this thickness an exact number of times, there will be waste, or at least the final plank will have less thickness; its correct measurement, however, in accordance with this reduced thickness, is included in the result of the calculation. For instance, in cutting inch-thick planks out of a log, if the last plank should be only a quarter of an inch thick, the calculation would give, for this thin slice, the value of only one quarter of a complete inch-thick plank.

II. Solid Measure.

Problem 1.

To find the cubic contents of a squared log of timber.

[A log of timber is said to be squared when its outer irregularities have been removed, and it is reduced to a piece with four flat sides.]

Rule I.—*When the four sides are rectangles.* Multiply the three dimensions,—length, breadth, and thickness (or rather the numbers expressing these) together: the product will be cubic contents.

Examples.

1. A log of timber is $26\frac{1}{2}$ feet long, $18\frac{1}{2}$ inches broad, and $14\frac{1}{2}$ inches thick: how many cubic feet are there in it?

	ft.	in.	
	26	6	
$18\frac{1}{2}$ in. =	1	6	6
	26	6	
	13	3	
	1	1	3
	40	10	3

	40	10	3		
$14\frac{1}{2}$ in. =	1	2	6		
	40	10	3		
	6	9	8	6	
	1	8	5	1	6
Ans.	49 Ft.	4'	4''	7'''	6''''

Or thus:

$26\frac{1}{2}$ ft. = 318 inches
Multiplied by $18\frac{1}{2}$

= 5883
Multiplied by $14\frac{1}{2}$

1728) $85303\frac{1}{2}$ (49 Ft.

Remainder $631\frac{1}{2}$ Inches.

In the first of these operations, the entire process is worked out in full: in the second, the details of the two multiplications and of the final division are suppressed, in order to save room. The former mode of working, by cross multiplication, we consider to be that which should, in general, be preferred: the successive steps of the work may, by help of the Pence Table, be very rapidly executed (see p. 11); and in the final result the several terms, proceeding from left to right, regularly descend by twelfths: thus, the result 49 ft. $4' \, 4'' \, 7''' \, 6''''$, above, indicates that the cubic content of the log is 49 cubic feet, 4 twelfths of 1 cubic foot, 4 twelfths of one twelfth, 7 cubic inches; and, finally, 6 twelfths of a cubic inch. We have not here given any distinctive names to these several denominations, and it is enough to know that the unit of any term is, in value, one-twelfth of the unit in the immediately preceding term: the first 4, after the 49 Ft., is $\frac{4}{12}$ths of a cubic foot; the next 4 is $\frac{4}{144}$ of a cubic foot; the 7 is $\frac{7}{1728}$ of a cubic foot; and the 6 is $\frac{6}{1728 \times 12}$ or $\frac{6}{20736}$ of a cubic foot. It is thus plain that the third term after the 49 denotes 7 cubic inches, the term following denoting 6 twelfths of a cubic inch, that is, $\frac{1}{2}$ a cubic inch.*

The strict accuracy with which this result is given is not absolutely necessary in the practical measurement of timber; yet it is important that the computer should know the exact amount of error committed by rejecting one or more of the final denominations in any result worked out, as in the above instance, with scrupulous accuracy. In order to do this, whatever be the dimensions of the log with which he is concerned, let him imagine a second log, of which each of the ends is just a square foot: then every unit in the first denomination of his result, after cubic feet, will represent a slice off the end of this second log, an inch thick; in the foregoing Example, the 4 represents a solid slice 4 inches thick. Every unit in the next term represents a marginal portion of the inch-thick slice, an inch wide; that is, a stick of wood a foot long and an inch square at the ends. Again:

* The computer, however, will perhaps prefer to distinguish these several denominations by dashes marked with the pen, writing the foregoing result thus, 49 Ft. $4' \, 4'' \, 7\frac{1}{2}$ In. We have marked them in this way in the answers to some of the questions which follow.

every unit in the next term denotes an inch cut off the end of this stick of wood, and is therefore 1 cubic inch; and each unit in the term next following denotes a twelfth of a cubic inch: the 6, in the case above, being, as before remarked, ½ of a cubic inch. For all practical purposes, fractions of a cubic inch, even in such costly wood as Spanish mahogany, may be disregarded, at least, whenever the fraction is below ½: when it is (as above) exactly ½, as also when it exceeds ½, it may be taken as an additional whole inch. If it be desired to express the result in cubic feet and inches only, —without the intermediate denominations—we shall have merely to multiply the first term, after Feet, by 12, taking in the next following term; then to multiply the product by 12, taking in the next term; that is, the Inches: thus, in the case worked above, multiplying the 4 by 12, and taking in the 4 next to it, we have 52; and then multiplying this by 12, and taking in the 7½ inches, we have 631½ Inches, agreeing with the result in the second operation; so that, in Feet and Inches, the cubic content of the log is 49 Ft. 631½ In.

2. How many cubic feet are there in a log of timber 12 ft. 9 in. long, 2 ft. 10 in. broad, and 2 ft. 1 in. thick?
   Ans. 75 Ft. 3' 1'' 6 In., or 75 Ft. 450 In.

3. How many cubic feet are there in a log of which the length, breadth, and thickness are respectively 23½ ft., 33 in., and 19 in.?
   Ans. 96 Ft. 1' 5'' 6 In.

[The single dash here marks 12ths of a Foot, and the double dash 144ths.]

RULE II.—*When the log regularly tapers from one end to the other.* Take the breadth and thickness at the middle of the log: these measurements will be the mean breadth and thickness; and these and the length multiplied together, as in Rule I., will give the cubic contents, nearly.

NOTE.—The dimensions at the middle of the log, that is, the mean breadth and thickness, may be found by taking half the sum of the breadths at the two ends for the mean breadth, and half the sum of the two thicknesses for the mean thickness.

4. Required the cubic contents of a log of which the length is 18 feet, and the mean breadth and thickness 18 inches and 10 inches?

This Example may be worked the more readily by proceeding agreeably to the second method of solving Ex. 1 (p. 142), but without taking the trouble of reducing the feet, in the length of the log, to inches; since here there is no fraction of a foot to be taken account of. If we thus omit to multiply this dimension by 12, we must expunge the factor 12 from the divisor 1728; that is, we must divide only by 144. Moreover, the given dimensions here happen to be such that all the work which is absolutely necessary may be executed mentally, without putting pen to paper: for the divisor 144 is readily seen to be 8 times 18, so that if the factor 18 be suppressed in the multiplication, the divisor, instead of 144, will be only 8: but the other two factors are 18 and 10; that is 9×2, and 5×2, the product of which is 45×4; and expunging this 4, as also the factor 4 in the 8, the divisor of the 45 is reduced to the number 2; and therefore the quotient is $22\frac{1}{2}$ Ft., the required cubic contents. And in this way, by expunging factors common to either multiplier and to the divisor, the work may, where such common factors exist, always be shortened. The following is an additional example.

5. Required the cubic contents of a log of which the length is 16 feet, and the mean breadth and thickness each 14 inches?
   Ans. $21\frac{7}{9}$ Ft.

6. Required the cubic contents of a log 18 feet long, the ends of which are squares, each side of one end being 18 inches, and each side of the other end 12 inches?
   Ans. $28\frac{1}{8}$ Ft.

It was stated in the Rule, that the cubic contents of a regularly tapering log, calculated in the foregoing manner, are not determined with strict mathematical accuracy, but only nearly. It is, however, the rule always employed in the timber trade, though the results it gives are usually somewhat short of the exact result; a sort of compensation being thus made for the departure of the shape of the log from that uniformity in breadth and thickness which it is desirable that a log of timber should have. The strictly correct rule for a tapering log, of which the ends, though unequal, are similar figures, is as follows:—

To the areas of the two ends add the square root of their

product: multiply the sum by the length of the log, and take one-third of the product for the cubic contents.

Thus, taking Ex. 6, above, we have—

For the areas of the two ends, $18^2 + 12^2 = 324 + 144 = 468$; and for the sq. root of their product, $\sqrt{(324 \times 144)} = 18 \times 12 = 216$. Then, by this Rule, $(468 + 216) \times 18 \div 3 = 684 \times 6 = 4104$; and this divided by 144 gives $28\frac{1}{2}$ Feet; while the answer above is only $28\frac{1}{8}$ Ft., the difference being $\frac{3}{8}$ of a cubic foot, which may be regarded as an allowance for the inequality in the two ends of the log.

Problem 2.

To find the cubic contents of a log of round or unsquared timber.

When a tree is lopped of its branches, and but roughly dressed, it is called round timber; and the circumference of it at any part is called the girt (or girth) at that part. When the tree tapers regularly, the girt in the middle, or half the sum of the girts at the two ends, is the mean girt; but it is often more satisfactory to girt the tree in several places, and to divide the sum of the several girts by the number of them for the mean girt. One-fourth of the mean girt is called the quarter-girt of the tree; and the Rule for the cubic contents is as follows.

Rule.—Multiply the number of inches in the quarter-girt by itself, and the product by the number of feet in the length: the result, divided by 144, will give the number of cubic feet in the tree.

Examples.

1. A tree is 24 feet long, its girt at the thicker end 14 feet, and at the thinner end 2 feet: what are its cubic contents?

   16 ft. $\div 2 = 8$ ft. $= 96$ in., the mean girt, therefore 24 in. = the quarter-girt; then by the Rule, $24 \times 24 \times 24 \div 144 = 2 \times 2 \times 24 = 96$ Ft.

In all cases where multiplications and divisions are concerned, the computer should be on the look out for factors common to multipliers and divisors, and suppress them *before* entering upon the numerical work: thus, in the

present example, an experienced calculator would not write down $24 \times 24 \times 24 \div 144$ at all; but, dismissing the factors 12, 12, which he would see at a glance to be common to the multipliers 24, 24, and to the divisor 144, would write merely the factors left.

2. A tree is $20\frac{1}{2}$ feet long, and its quarter-girt is $10\frac{1}{4}$ inches: required its cubic contents?

The operations to be performed are thus indicated, viz., $10\frac{1}{4} \times 10\frac{1}{4} \times 20\frac{1}{2} \div 144$, and these we shall execute as in the margin, disregarding the multiplication of $\frac{1}{4}$ by $\frac{1}{4}$; since, as we shall presently see, the fraction arising from this multiplication is of no practical consequence. The result of the operation shows that the contents of the tree are 14 cubic feet and $11\frac{3}{8}$ twelfths of a cubic foot besides.

$$\begin{array}{rl}
 & 10\frac{1}{4} \\
 & 10\frac{1}{4} \\ \hline
 & 100 \\
 & 2\frac{1}{2} \\
 & 2\frac{1}{2} \\ \hline
 & 105 \\
 & 20\frac{1}{2} \\ \hline
 & 2100 \\
 & 52\frac{1}{2} \\ \hline
144) & 2152\frac{1}{2} \\ \hline
 & 14 \text{ Ft.} \\ \hline
12) & 136\frac{1}{2} \text{ Rem.} \\ \hline
 & 11\frac{3}{8} \\ \hline
\end{array}$$

Let us now estimate the amount of error in this conclusion, arising from our neglect of the $\frac{1}{4} \times \frac{1}{4}$, or $\frac{1}{16}$, which, in strictness, ought to have entered the first product, 105, the correct product being $105\frac{1}{16}$. The omission of this fraction causes an error of $\frac{1}{16} \times 20\frac{1}{2}$, that is, of $\frac{41}{32}$, in the next product, which, accurately, is $2152\frac{1}{2} + \frac{41}{32}$; so that, in addition to the 14 Ft. $+ \dfrac{136\frac{1}{2}}{144}$ Ft., we ought to have had $\dfrac{41}{32 \times 144}$ Ft. besides. Now this fraction is obviously the sum of the two fractions $\dfrac{32}{32 \times 144}$ and $\dfrac{9}{32 \times 144}$; that is, it is equal to $\frac{1}{144} + \dfrac{9}{32 \times 144}$, and therefore *less* than $\frac{1}{144} + \frac{1}{3}$ of $\frac{1}{144}$. Hence, referring to our standard log, or log of reference, adverted to at p. 143, and which is a log 1 foot in breadth and in thickness throughout, it follows that the sacrifice of timber, in the case before us, is less than a slice off the end of that log, $\frac{1}{12}$ of an inch plus $\frac{1}{3}$ of $\frac{1}{12}$ in thickness; that is to say, a slice scarcely the thickness of a shilling, which, of course, in a log of rough timber, is of no moment whatever.

We have entered into these considerations here, in order that the timber-measurer may see how unnecessary it is, in multiplying the quarter-girt by itself, that such fractions of a square inch should be computed, and retained in the product. He will perceive, from the above, that in general, if this product be determined to the nearest inch only—in excess of the exact product, if the fraction be equal to or greater than ½, and in defect of the exact product, if the fraction be less than ½—no appreciable error in the cubic contents of the log will be committed.

Moreover, there is an additional reason for disregarding these niceties in calculating the cubic measurement of rough timber. The quarter-girt Rule, given above, however strictly followed, does not itself give the exact cubic measurement, nor is it intended to do so: the true result is about one-fourth more than the result arrived at by the Rule, as may be proved from geometrical considerations; but it is the custom of the trade to make this allowance of one-fourth for the waste (as timber), in squaring the rough log. The greatest squared log that can be cut out of any portion of a rough round tree, is that of which the breadth and thickness are equal; that is, the cross-section must throughout be a square; for any other four-sided shape, the timber, in the squared log, will be less; and this fact it is well should be generally known: the allowance for waste presumes that the squaring is thus economically executed. This circumstance—of the trade-rule giving the cubic measurement of a rough log one-fourth less than the actual number of cubic feet in it—satisfactorily explains why it is that, in Tables of the bulks of different materials which go to a *load*, we find

"40 cubic feet of round or rough timber = 1 load."
and "50 feet of squared timber = 1 load."

The 40 cubic feet of round or rough timber, as calculated per Rule, is in reality 40 feet plus one-fourth of 40 feet; that is, it is 50 feet. (See following TABLE.)

3. A piece of round timber is 9 feet 6 inches long, and its quarter-girt 42 inches: how many cubic feet are there in it? *Ans.* 116⅜ Ft.
4. The average girt of a round log is 6 feet 3 inches, and its length is 12 feet: how many cubic feet are there in it? *Ans.* 29¼ Ft.

NOTE.—If the tree taper very irregularly, it is best to divide it into several lengths, and to find the cubic contents of each portion separately.

Timber Table.

40 cubic feet of round timber make				1 load.
50 „ squared timber „				1 load.
600 square feet of inch-thick plank „				1 load.
400 „	$1\frac{1}{2}$ in.	„	„	1 load.
300 „	2 in.	„	„	1 load.
200 „	3 in.	„	„	1 load.
150 „	4 in.	„	„	1 load.

Average Weight of a Cubic Foot, in lbs. Avoirdupois.

Name of Wood.	Weight in lbs.	Name of Wood.	Weight in lbs.
Cork	15	Cherry-tree	44·68
Poplar	23·94	Teak	46·56
Larch	34	Maple and Riga Fir	46·87
Elm	34·75	Ash and Dantzic Oak	47·5
Honduras Mahogany	35	Dutch Yew	49·25
Willow	36·56	Apple-tree	49·56
Cedar	37·25	Alder	50
Pitch Pine	41·25	Spanish Yew	50·44
Pear-tree	41·31	Spanish Mahogany	53·25
Walnut	41·94	Canada Oak	54·5
Mar Forest Fir	43·37	French Box	57
Elder-tree	43·44	English Oak	60·62
Beech	43·5	Ditto 60 years old	73·12

[We see by this Table how inaccurate it is to call, indiscriminately, a load of wood (50 cubic feet) "a ton."]

From the foregoing Table, the number of cubic feet in a misshapen log, or block of wood, may be ascertained from knowing the weight of the mass; and conversely, the cubic contents being known, the weight may be found, as in the two problems following.

Problem 1.

The weight being given, to find the cubic contents of a piece of timber.

Rule.—Divide the number of lbs. in the given weight by the number of lbs. in a cubic foot, as given in the Table; and the quotient will be the number of cubic feet in the piece.

EXAMPLES.

1. How many cubic feet are there in a ton of Honduras Mahogany? By the Table, a cubic foot weighs 35 lbs., and 2240 is the number of lbs. in a ton: hence by the Rule, $2240 \div 35 = 64$, the number of cubic feet in a ton. If the wood had been English Oak, the work would have been $2240 \div 60{\cdot}62 = 36{\cdot}95$, the number of cubic feet in a ton of English Oak.
2. How many cubic feet are there in 7 cwt. 12 lbs. of Spanish Mahogany? 7 cwt. 12 lbs. = 796 lbs.; then, by the Rule, $796 \div 53{\cdot}25 = 14{\cdot}94$, the number of cubic feet. *Ans.*
3. How many cubic feet are there in a ton of Beech? *Ans.* 51·49 cubic feet.
4. How many cubic feet are there in a ton of Riga Fir? *Ans.* 47·76 cubic feet.
5. How many cubic feet are there in a mass of English Oak weighing 11 cwt? *Ans.* 20·32 cubic feet.

[It will of course be understood that the above are the average, or medium results, in each case; for the same weight of different specimens may vary in bulk; and conversely.]

PROBLEM 2. (CONVERSE OF PROB. 1.)

The bulk of a piece of timber being given, to find its weight.

RULE.—Multiply the cubic contents of the piece by the corresponding tabular number; and the product will be the number of lbs. weight.

EXAMPLES.

1. What is the weight of a log of Larch 14 feet long, $2\frac{1}{2}$ feet broad, and $1\frac{1}{4}$ feet thick?
   $2\frac{1}{2} \times 14 = 35$; and $35 \times 1{\cdot}25 = 43{\cdot}75$ Ft. the cubic contents.
   Then, $43{\cdot}75 \times 34 = 1487{\cdot}5$ lbs. = 13 cwt. 1 qr. $3\frac{1}{2}$ lbs., *Ans.*
2. What is the weight of a log of Honduras Mahogany, of which the contents are 64 cubic feet? *Ans.* 1 ton.
3. What is the weight of a piece of Spanish Mahogany, of which the contents are 14·94 cubic feet? *Ans.* 7 cwt. 12 lbs.
4. What is the weight of a log of Pitch Pine, 24 feet long, 3 feet broad, and $2\frac{1}{2}$ feet thick? *Ans.* 3 tons 6 cwt. 23 lbs.

[From the numbers in the preceding Table may easily be deduced the *Specific Gravity* of any of the woods there registered. By the specific gravity of any substance, is meant

the ratio of the weight of any bulk of it, to the weight of an equal bulk of distilled water, when at the temperature of 60° of Fahrenheit's thermometer. At this temperature, a cubic foot of water weighs 1000 ounces avoirdupois, that is, $62\frac{1}{2}$ lbs. If therefore anyone of the numbers in the Table be divided by $62\frac{1}{2}$, the quotient will be the ratio alluded to; that is, the specific gravity of the wood which that number stands against. For example, take larch: then $\frac{34}{62\frac{1}{2}} = \frac{68}{125}$ $= \cdot544$, the specific gravity of larch.]

CALCULATIONS USEFUL IN THE WORK OF ARTIFICERS.

By the term "Artificers" is to be understood those classes of workmen whose workmanship, as well as the materials on which they operate, is estimated by measurement. Such are Carpenters, Joiners, Masons, &c.

I.—CARPENTERS' AND JOINERS' WORK.

This kind of work, as called into requisition in a building, comprehends flooring, partitioning, wainscoting, roofing, &c. The workmanship is generally estimated in square measure, though cornices, mouldings, and the like ornamental work, are usually measured by the lineal foot. Flooring and partitioning, and other boarding of much extent, are measured by the square of 100 superficial feet; that is, a square of boarding, whether floor, or wainscot, or partition, &c., is 100 superficial feet.

A SQUARE OF BOARDING REQUIRES,

With boards 10 *feet long,*

24 boards, 5 inches broad.	15 boards, 8 inches broad.
20 „ 6 „	12 „ 10 „

If the boards be 7 inches broad, there must be 17 of them, and a slip 1 inch broad off the whole length of another board besides, to make up the square; the measure of this additional piece being 120 square inches, or $\frac{5}{6}$ square foot. If the boards be 9 inches broad, there must be 13 of them, with an additional slip 3 inches broad besides; the measure of this slip being $2\frac{1}{2}$ square feet.

With boards 12 feet long.

20 boards,	5 in. broad.		12 boards,	8 in. broad	+ 4 sq. ft.
16 „	6 „	+ 4 sq. ft.	11 „	9 „	+ 1 sq. ft.
14 „	7 „	+ 2 sq. ft.	10 „	10 „	

*** The additional square feet here are the measure of a slip so many inches broad and 12 feet long.

The reader may readily satisfy himself that the number of boards specified above is, in each case, such as to make up exactly 100 superficial feet; thus, the number of ten-foot boards, laid side by side, must make an entire breadth of boarding of 10 feet, or 120 inches, in order that the whole surface may measure 100 square feet; and it is easy to see that this is the breadth reached by using the number of boards stated in the Table. When the breadth of board is 7 inches or 9 inches, a fraction of a board, or a slip off the whole length of it, is necessary to complete the square; in the former case, the slip must be 1 inch broad, in the latter case, 3 inches, because $17 \times 7 = 119$ only, instead of 120, and $13 \times 9 = 117$ only, instead of 120. A slip an inch broad, off a board 10 feet or 120 inches long, measures, of course, 120 square inches.

With boards 12 feet long, the entire breadth, to complete 100 square feet, must be 100 inches, that is $8\frac{1}{3}$ feet: and every inch of breadth, cut off the entire length of such a board, is a slip measuring 144 square inches, or 1 square foot: the correctness of the statements in the Table are thus obvious. But if the boards are rough, and unprepared for wrought flooring, then an allowance must be made for the loss of breadth in planing the edges; thus, while twelve and a half, 12-feet boards, 8 inches broad, suffice for a square of rough flooring, thirteen are allowed for a square of wrought flooring; and for rough boards, 9 inches broad, $11\frac{1}{2}$ are allowed for a square of wrought flooring; while of 15-feet battens, 7 inches broad, fifteen are considered necessary, if the edges are to be planed. The narrower the board the greater, of course, is the waste in planing.

There are what are called "Standard Measurements" for Deals and Battens, these latter being only a narrower kind of deals. The London and Dublin standard deal is 12 feet long, 9 inches broad, and 3 inches thick; while the batten (of whatever length) is 7 inches broad and 3 inches

c. When the breadth and thickness—one or both—r from these dimensions, the piece is called simply a d, or a plank; but when a three-inch deal, or batten, wn into thinner boards for flooring, partitioning, &c., board is called a leaf.

ıe standard for deals differs in different countries; the ıipal are as follows:—

Standard Deals.

	Length.	Breadth.	Thickness.	
sburg . . .	12 ft.	11 in.	$1\frac{1}{2}$ in.	$= 16\frac{1}{2}$ sq. ft., 1 in. thick.
tiania . . .	11	9	$1\frac{1}{4}$	$= 10\frac{5}{16}$ " "
ion and Dublin	12	9	3	$= 27$ " "
ec [100] . .	12	11	$2\frac{1}{2}$	$= 27\frac{1}{2}$ " "

ı the Timber-market, 120 deals go to the hundred, pt the Quebec 100; and, consequently, in a Petersburg ıdred there are 1980 superficial feet, 1 inch thick, or 165 c feet; in a Christiania Hundred, $1237\frac{1}{2}$ superficial feet, ch thick, or $103\frac{1}{8}$ cubic feet; in a London and Dublin ıdred, 3240 superficial feet, 1 inch thick, or 270 cubic ; and in a Quebec Hundred, 2750 superficial feet, 1 inch k, or $229\frac{1}{6}$ cubic feet.

EXAMPLES.

ı piece of boarding measures 96 ft. 3 in. by 21 ft. 3 in.: how many squares are there in it?

om the annexed work, it appears that there are sq. ft., 3 twelfths of a sq. ft., and 9 sq. in.; that is y, 2045 Ft. 45 In.: so that, dividing the number of re feet by 100, the answer is 20 Squares, 45 Ft., 45 In.

ft.	in.	
96	3	
21	3	
2021	3	
24	0	9
20,45	3	9

f a floor be 57 ft. 3 in. by 28 ft. 6 in., how many squares are there in it? [See the work below.]

the 7, in the second place, represents so many twelfths of a square and the 6, in the next place, so many square inches, nswer is 16 Squares, 31 Ft., 90 In. $= 16\frac{8}{25}$ sq. nearly.

ft.	in.	
57	3	
28	6	
1603	0	
28	7	6
16,31	7	6

n all calculations of this kind care must be n to avoid the not uncommon error of re-ling the second term in the result as so many ıre inches, instead of so many twelfths of a ıre foot. Each of these twelfths is, of course, ıches, not 1 inch; it is the third term that tes square inches: the 7 above represents 84 square es.]

3. A partition measures 91 ft. 9 in. by 11 ft. 3 in.: how many squares are there in it? *Ans.* 10 Squares, 32 Ft., 27 In.
4. A wainscoted room is 16 feet 3 in. high, and 137 ft. 6 in. in compass: how many squares are there in it?
   Ans. 22 Squares, 34 Ft., 54 In.
5. A piece of boarding measures 36 ft. 4 in. by 12 ft. 3 in.: what did it cost (material and workmanship), at the rate of £6 15*s.* per square? *Ans.* £30 0*s.* 10½*d.*

It may be well that we should exhibit the work of this example at length.

ft.	in.	
36	4	
12	3	
436	0	
9	1	0
4,45	1 = 4 Sq. 45 Ft. 12 In.	

	£	*s.*	*d.*	
	6	15	0	
			4	
Value of 4 Sqs. =	27	0	0	
,, 25 Ft. =	1	13	9	= ¼ of £6 15*s.*
,, 20 Ft. =	1	7	0	= ⅕ ,,
	£30	0*s.*	9*d.*	

[45 Ft. = 25 Ft. + 20 Ft. = ¼ of 100 + ⅕ of 100.]

The 12 Inches are still to be valued; that is, the twelfth part of 1 Ft. The value of 1 Ft. being the 20th part of 27*s.*, we have to divide this by 20 and 12, or to take the 20th part of 27 pence, which may be regarded as 1½*d.*; so that the answer to the question is £30 0*s.* 10½*d.* The odd halfpenny would, of course, be disregarded. But the price of a square of boarding, or flooring, &c., being given, the price of any number of squares, and parts of a square, may be readily calculated by the following Rule.

Problem 1.

The price of a square being given, to find the price of any number of squares and fractions of a square.

Rule.—Regard each Square as £5, and therefore each Ft. at 1*s.*, and each twelfth of 1 Ft. as 1*d.* We shall thus have the price of the whole at the rate of £5 per square: then take parts for the difference between £5 and the given price per square.

Thus, returning to the Example above, we have:—

	£	*s.*	*d.*		£	*s.*
For 4 Sq. $45\frac{1}{12}$ Ft.	22	5	1	= price at	5	0
⅕ of the above	4	9	0	= ,,	1	0
½ of last sum	2	4	6	= ,,	0	10
¼ ,,	1	2	3	= ,,	0	5
	£30	0*s.*	10*d.*	,,	£6	15*s.*

We have here disregarded the fractions of $1d.$; they are as follows: $\frac{1}{5}d. + \frac{1}{10}d. + \frac{1}{20}d. = \frac{4 + 2 + 1}{20}d. = \frac{7}{20}d.$, the same as before.

2. The dimensions of a floor are 53 ft. 6 in. by 27 ft. 9 in.: required the cost, at £3 15*s.* per square? *Ans.* £95 15*s.* 11½*d.*
3. If a floor be 57 ft. 3 in. by 28 ft. 6 in., what will be the cost of the boarding, at £1 13*s.* 4*d.* per square? *Ans.* £27 3*s.* 10½*d.*
4. What will be the cost of a boarded floor, measuring 86 ft. 10 in. by 28 ft. 6 in., at the rate of 18*s.* per square? *Ans.* £37 16*s.*

NOTE.—In the foregoing examples we have considered the flooring to be boarded flooring; but it may be proper here to remind the reader that, previously to the boarding, a naked flooring, as it is called, is constructed to support the boards. This is a framework, the component pieces of which are called joists and girders, which are partially inserted in the brickwork or masonry. The squares in a naked flooring are computed in exactly the same way as the squares in a boarded flooring,—the measurements of length and breadth being extended to the extreme limits of the timber, including the insertions in the walls, though these are out of sight. Roofing, also, is computed in like manner; the girt of the roof, measured with a string from the extreme end of one rafter, over the ridge, to the extreme end of the opposite rafter, is taken for the breadth, and this is multiplied by the length, and the number of squares of roofing found, just as if these were the dimensions of a boarded floor.

We here, however, consider the roof to present but two sloping faces, terminating in a single ridge. But in many structures the roof presents four faces or slopes, the opposite pair being usually symmetrical. Every such face is measured and computed exactly as we measure and compute the surface of a tapering board or plank (p. 139), that is, the two parallel lengths are added together—the base of the slope and the upper parallel ridge—and half their sum is multiplied by the distance between them; this being the length of a string stretched from one to the other. Should either face terminate in a point, instead of in a ridge, then merely half the base of the triangle is to be multiplied by the distance of it from the vertex, or terminating point; for then there is no length of upper ridge to be added. The surfaces of all the faces being added together, the sum will be the surface-measure, in square feet, of the whole roof. If the faces terminate in a flat, the surface of this flat is of course to be added.

PROBLEM 2.

The price per Hundred (120) *deals being given, to find the price per single deal.*

RULE.—Twice the number of £'s in the price of a Hundred will be the number of pence in the price per single deal.

Examples.

1. At £12 per Hundred, what is the price of a single deal?
   $12 \times 2 = 24$ pence; therefore the price is 2*s*.
2. At £14 5*s*. per Hundred, what is the price per deal?
   $14\frac{1}{4} \times 2 = 28\frac{1}{2}$ pence $=$ 2*s*. $4\frac{1}{2}$*d*. *Ans.*

The reason of the Rule is pretty obvious: there are 120 twopences in £1; so that, at £1 per Hundred, each deal would cost 2*d*.; hence as many £'s as there are in the cost of a Hundred, so many twopences must there be in the cost of one.

3. At £15 5*s*. per Hundred, what is the price per deal? *Ans.* 2*s*. $6\frac{1}{2}$*d*.
4. At £16 15*s*. per Hundred, what is the price per deal? *Ans.* 2*s*. $9\frac{1}{2}$*d*.

Problem 3. (Converse of Prob. 2.)

The price of a single deal being given, to find the price of a Hundred (120).

Rule.—Half the number of pence in the price of one will be the number of £'s in the price of 120.

This Rule is an obvious inference from that of last problem.

Examples.

1. If a single deal cost 2*s*. 10*d*., what is the price of 120?
   2*s*. 10*d*. $=$ 34 pence; therefore the price of 120 is £17.
2. If a single deal cost 2*s*. $6\frac{1}{2}$*d*., what will 120 cost? *Ans.* £15 5*s*.
3. If a single deal cost 2*s*. 3*d*., what will 120 cost? *Ans.* £13 10*s*.

Problem 4.

The price per running foot (foot of the length) of a standard deal being given, to find the price of a Hundred; and conversely.

Rule.—1. Six times the number of pence in the price of a running foot will be the number of £'s in the price of 120 standard deals.

For as many pence as 1 foot costs, so many shillings will 12 feet, or a whole deal cost. But there are 120 deals; and $120 = 6 \times 20$: hence, if we multiply the before-mentioned number of shillings (that is, the given number of pence) by 6, the result must be the number of £'s in the cost of the whole 120.

2. *Conversely.* Divide the number of £'s in the cost of 120 standard deals, by 6, and the result will be the number of pence per running foot.

These two Rules may be expressed a little differently: thus, for the first we may say,—Regard the pence in the given price as so many £'s, and multiply by 6: and for the second,—Regard the £'s in the given price as so many pence, and divide by 6. We shall here work by the Rules in this form.

EXAMPLES.

1. What is the price of 120 standard deals, at $3\frac{1}{2}d.$ per running foot; and what the price at $2\frac{3}{4}d.$?

   1st. $£3\frac{1}{2} \times 6 = £21.$ *Ans.* 2nd. $£2\frac{3}{4} \times 6 = £2\ 15s. \times 6 = £16\ 10s.$ *Ans.*

2. At £16 per Hundred, what is the price of a running foot; and what the price at £13 10*s.*?

   1st. $16d. \div 6 = 2\frac{2}{3}d.$ *Ans.* 2nd. $13\frac{1}{2}d. \div 6 = 27d. \div 12 = 2\frac{1}{4}d.$ *Ans.*

3. Required the price of 120 standard deals, at $2\frac{1}{4}d.$ per running foot? *Ans.* £13 10*s.*

4. If 120 standard deals cost £16 15*s.*, what is the cost per running foot? *Ans.* $2\frac{3}{4}d.$

NOTE.—A 12-foot board, whether a standard deal or not, always contains as many superficial feet as there are inches in its breadth; for the surface, in square feet, is 12 ft. multiplied by the 12th part of the number of inches in the breadth; and 12 times the 12th part of any number is that number itself. The value of any portion of breadth cut off, and running the whole length of such a board, is therefore easily found; thus,—As the whole breadth is to the breadth of the slip cut off, so is the price of the board to the value of the part taken away. For example: From a 12-foot deal, 9 in. broad, a slip, 2 in. broad, is cut off; what is the value of it, at 2*s.* 6*d.* for the deal?

$$\text{The value is } \tfrac{2}{9} \text{ of } 2s.\ 6d. = 5s. \div 9 = 6\tfrac{2}{3}d.$$

Again:—If 2*s.* 6*d.* be charged for a 12-foot batten, 7 inches broad, cut off a board of the same length, 11 inches broad, what is the entire board valued at? The value is $\frac{11}{7}$ of $2s.\ 6d. = 27s.\ 6d. \div 7 = 3s.\ 11\frac{1}{7}d.$*

* For much practical information respecting the purchasing and measurement of Timber and Deals, see "The Timber Importer's and Timber Merchant's Guide," by Richard E. Grandy: published by Crosby Lockwood and Co.

II.—Bricklayers' Work.

Brickwork is usually measured by either the square yard, or the square pole, or rod, or perch. As the linear English rod, or perch, is $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, the square rod or perch contains $30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet

The surface of the brickwork being measured in this way, the unit or standard of thickness is fixed at a brick and a half, in addition to the mortar; so that if there be more or less than a brick and a half in the thickness of a wall, the number of square yards or rods is computed accordingly. Thus, if the wall be 2 bricks thick, its thickness is $1\frac{1}{3}$ of the standard thickness; if 3 bricks thick, twice standard thickness; if 1 brick thick, $\frac{2}{3}$ of standard thickness, and so on.

Problem 1.

The length and height of a wall being given, and the number of half bricks in its thickness, to find how many yards or rods of brickwork it contains.

Rule—1. Multiply the length and height together, for the number of square feet in the surface of the wall.

2. Multiply this number of square feet by the number of half bricks in the thickness, and divide the product by 3: the quotient will be the number of square feet a brick and a half thick. Divide again by 9, to bring these feet to square yards, or by $272\frac{1}{4}$ to bring them to square rods or perches.

Examples.

1. How many sq. rods of brickwork are there in a wall, of which the length is 57 ft. 3 in., and the height 24 ft. 6 in.; the wall being $2\frac{1}{2}$ bricks thick?

	ft.	in.		
	57	3		
	24	6		
	1374	0		
	28	7	6	
	1402	7	6	= area of surface of wall.
			5	= half-bricks in thickness.
3)	7013	1	6	
$272\frac{1}{4}$)	2337	8	6	(8 sq. rods, $17\frac{2}{3}$ sq. yards; the *Ans.*
	2178			
9)	159			
	$17\frac{6}{9} = 17\frac{2}{3}$ sq. yards.			

The $8\frac{1}{2}$ twelfths of a sq. foot have not been divided by the 9, and so converted into a fraction of a yard, as the result would be insignificant; but instead of entirely rejecting the $8\frac{1}{2}'$ as here, the small error would have been still less if we had replaced it by an additional foot, making the dividend 2338, and therefore the overplus $17\frac{1}{9}$ sq. yards; which errs less in excess than $17\frac{8}{9}$ errs in defect; the former measure being only $3\frac{1}{2}'$ in excess, while the latter is $8\frac{1}{2}'$ in defect.

The calculation may be performed a little more expeditiously in the manner here annexed. The number 2338 is the measure of the brickwork to the nearest sq. foot; and the measure in sq. rods is obtained by dividing this number by $272\frac{1}{4}$, as above. Whenever the odd inches in the linear measurement amount to half or a quarter of a foot, the superficial contents may often be most readily computed in this way.

$$\begin{array}{rr}
 & 57\frac{1}{4} \\
 & 24\frac{1}{2} \\
\hline
 & 234 \\
 & 114 \\
 & 28\frac{5}{8} \\
\hline
 & 1402\frac{5}{8} \\
 & 5 \\
\hline
3) & 7013\frac{1}{8} \\
\hline
 & 2338
\end{array}$$

2. How many square yards of brickwork are there in a wall 75 ft. long, and 15 ft. 9 in. high, the wall being 3 bricks thick? *Ans.* 262 Yds. $4\frac{1}{2}$ Ft.

[When the thickness is 3 bricks, instead of multiplying by the number of half-bricks (6), and dividing by 3, we need merely multiply by 2; and when the thickness is $4\frac{1}{2}$ bricks, we have only to multiply by 3.]

3. A garden, 160 feet broad, contains exactly one acre: what would be the expense of inclosing it with a brick wall $10\frac{1}{2}$ ft. high and $2\frac{1}{2}$ bricks thick, allowing the work to be at the rate of 5*s.* $7\frac{1}{2}$*d.* per sq. yard, of standard thickness; deductions being made for two doors, each 6 ft. 9 in. by 4 ft., and for a gateway, the height of the wall, and 11 ft. wide? *Ans.* £463 18*s.* $10\frac{3}{4}$*d.*

It has been found that 272 feet of superficial area of brickwork, a brick and a half thick, require 4500 bricks, making due allowance for waste: hence the number of bricks necessary for every square foot of this surface will be found by dividing 4500 by 272. Now

$$4500 \div 272 = 16{\cdot}544117\ldots$$

Consequently two-thirds of this will be the number of bricks required for each square foot of the work, when it is only 1 brick thick; four-thirds, when it is 2 bricks thick; five-thirds when it is $2\frac{1}{2}$ bricks thick; and so on; and in this way is the following Table constructed.

[It will be noticed that the number of bricks in a square foot is in no case exact; and that the decimal parts are given to three places. The object of so doing is to make the table equally useful for large as for small surfaces, as will be seen by the examples.]

Table

Showing the number of bricks required for walls of different thicknesses.

Superficial area in sq. ft.	Number of bricks required.				
	1 brick.	1½ bricks.	2 bricks.	2½ bricks.	3 bricks.
1	11·029..	16·544..	22·059	27·574	33·088
2	22·059	33·088..	44·118	55·147..	66·176..
3	33·088..	49·632..	66·176..	82·721	99·265
4	44·118	66·176..	88·235..	110·294..	132·353
5	55·147..	82·721	110·294..	137·868	165·441
6	66·176..	99·265	132·353	165·441..	198·529..
7	77·206	115·809	154·412	193·015	231·618
8	88·235..	132·353	176·471	220·589	264·706
9	99·265	148·897..	198·529..	248·162	297·794

The practical use of this Table will be sufficiently seen from the following example.

4. Required the number of bricks necessary to build a wall 2½ bricks thick, the superficial area of the face of the wall being 2346 feet?

Number of bricks for 2000 sq. ft. = 1000 times the number for 2 sq. ft. . . . . } = 55147

Number of bricks for 300 sq. ft. = 100 times the number for 3 sq. ft. . . . . } = 8272·1

Number of bricks for 40 sq. ft. = 10 times the number for 4 sq. ft. . . . . } = 1102·94

Number of bricks for 6 sq. ft. = 6 times the number for 1 sq. ft. . . . . } = 165·44

The number of bricks required = 64687·48

Hence 64688 bricks will be necessary.

Note.—Brick and mortar work is frequently estimated not by the square rod or perch, which is a square surface of brickwork 16½ feet each way and 1½ bricks thick, but by what is called the "standard perch," which is a mass of brickwork, in surface 16½ feet by 1 foot, and (bricks and mortar) 14 inches thick. The cubic contents of the standard perch are therefore 16½ ft. $\times$ 1 $\times$ 1⅙ = 19¼ cubic feet, and consequently by this number must the cubic contents of the brickwork be divided to get the number of standard perches in it. If, however, the work be a brick and a half thick, that is (including the *mortar* joint), 14 inches, then it will be sufficient to compute the surface *only of the brick*work, and to divide the result by 16½, instead of by 19¼;

because in finding the cubic contents we should multiply by 14 inches, or $1\frac{1}{6}$ ft., and then afterwards divide by $16\frac{1}{2} \times 1\frac{1}{6}$: it is better therefore to omit the $1\frac{1}{6}$ both as multiplier and divisor. We thus have the following Rule to find the number of "standard perches" in a piece of brickwork.

Problem 2.

To find the number of standard perches in a piece of brickwork.

Rule—1. *When the work is* 14 *inches thick.* Find the number of square feet in the surface, and divide that number by $16\frac{1}{2}$; the result will be the number of standard perches.

2. *When the work is of any other thickness.* Find the number of cubic feet in the entire mass, and divide by $19\frac{1}{4}$; the result will be the number of standard perches.

Examples.

1. How many standard perches of brickwork are there in a wall 40 ft. long, $10\frac{1}{2}$ ft. high, and 14 in. thick?

$$
\begin{array}{rl}
 & 10\frac{1}{2} \\
 & 40 \\
\hline
16\frac{1}{2}) & 420 \text{ sq. ft. of surface.} \\
\hline
 & 25\frac{15}{33} = 25\frac{5}{11} \text{ standard perches. } \textit{Ans.}
\end{array}
$$

2. How many standard perches of brickwork are there in the front wall of a house 25 ft. wide, 30 ft. high, and 18 in. thick; the following openings (or "opes," as workmen call them) being deducted,—namely,—

2 windows	5 ft. by $3\frac{1}{2}$ ft.	=	35 sq. ft.
2 ,,	6 ft. by 4 ft.	=	48 ,,
2 ,,	$4\frac{1}{2}$ ft. by 3 ft.	=	27 ,,
1 door	7 ft. by $3\frac{1}{2}$ ft.	=	$24\frac{1}{2}$,,
Total deduction for "opes"		=	$134\frac{1}{2}$ sq. ft.

One of the three dimensions to be multiplied together, namely, the $1\frac{1}{2}$ ft. of thickness, we shall replace by the number 6, which is 4 times

$1\frac{1}{2}$. By so doing we shall simplify the work, replacing the divisor $19\frac{1}{4}$ by 77, which is 4 times $19\frac{1}{4}$: the calculation will then stand as follows:—

$$
\begin{array}{rl}
25 \times 30 = & 750 \text{ sq. ft. of wall.} \\
\text{Deduct} & 134\tfrac{1}{2} \text{ for "opes."} \\
\hline
& 615\tfrac{1}{2} \text{ sq. ft. of actual brickwork.} \\
& \quad 6 = 4 \text{ times } 1\tfrac{1}{2}. \\
\hline
4 \text{ times } 19\tfrac{1}{4} = 77) & 3693 \; (47\tfrac{74}{77} \text{ standard perches.} \\
& 308 \\
\hline
& \;613 \\
& \;539 \\
\hline
& \;\;74
\end{array}
$$

It thus appears that there are 48 standard perches of brickwork, very nearly.

It should be mentioned that the mass of brickwork in an Irish perch differs from that in a standard perch. The linear measure of the Irish perch is 7 yards, instead of $5\frac{1}{2}$ yards; and the Irish perch of brickwork is 21 ft. $\times$ 1 $\times$ $1\frac{1}{6} = 24\frac{1}{2}$ cubic ft., per Irish perch.

3. How many Irish perches of brickwork are there in a wall 60 feet long, 8 ft. 4 in. high, and 18 in. thick?

[To avoid fractions, we shall multiply the thickness ($1\frac{1}{2}$ ft.) by 2, and also the divisor $24\frac{1}{2}$ by 2.]

$8\frac{1}{3} \times 60 = 500$; and 3 times this is 1500; then $49)1500(30\frac{30}{49}$ so that the wall contains $30\frac{30}{49}$ Irish perches; that is, $30\frac{5}{8}$ Irish perches very nearly.

4. A piece of brickwork is 66 ft. long, 20 ft. 6 in. high, and 28 in. thick: required 1st, the number of standard perches in it; and 2nd, the number of Irish perches?

Ans. 164 standard, and $128\frac{6}{7}$ Irish perches.

NOTE.—A standard perch of brickwork is to an Irish perch as 11 is to 14; and in this proportion is the number of Irish to the number of standard perches in any piece of brickwork.

[Although a standard, whether of measurement or of anything else, ought to be fixed and definite, yet builders recognize two standard perches of brickwork; namely, the standard perch for 14-inch work, and the standard perch for *9-inch work*; the superficial dimensions, that is, the area of

the face of the brickwork, being the same in both, $16\frac{1}{2}$ sq. feet. The standard perch of 9-inch work contains

$$16\tfrac{1}{2} \times \tfrac{9}{12} = 16\tfrac{1}{2} \times \tfrac{3}{4} = 12\tfrac{3}{8} \text{ cubic feet};$$

so that allowing 9 inches for the standard thickness, the number of cubic feet in the wall must be divided by $12\frac{3}{8}$ to get the number of standard perches, of that thickness. It is easy to see that 14 in. : 9 :: $19\frac{1}{4}$: $12\frac{3}{8}$; as it ought to be.]

III.—Masons' Work.

The calculations required for the measurement of masonry are so much like those for bricklayer's work, that but little in addition to what has been explained in the preceding article need be said here. As in brickwork, so in masonry, the material and workmanship are estimated either by the superficial foot, the cubic foot or yard, or the standard perch. This last measure however differs from that employed in brickwork: the standard perch for masonry being $16\frac{1}{2}$ ft. $\times$ 1 ft. $\times$ $1\frac{1}{2}$ ft. $= 24\frac{3}{4}$ cubic feet*; and for the Irish perch, 21 ft. $\times$ 1 ft. $\times$ $1\frac{1}{2}$ ft. $= 31\frac{1}{2}$ cubic feet.

About 16 cubic feet of Portland stone weigh one ton; about 17 of Bath stone; $13\frac{1}{2}$ of Granite; and, at a medium, 13 of Marble.

As the cubic contents of every rectangular mass, of whatever material, are computed in one uniform manner, namely by multiplying the three dimensions—length, breadth and thickness—together, special rules for the purpose in the case of stone would be unnecessary: the method of proceeding will sufficiently appear from the subjoined examples. We may observe here, however, that in computing the superficial workmanship the dimensions are taken by pressing the measuring tape or string in and around every place which the tool touches. It is by superficial measure that chimney-pieces, pavements, and slabs are estimated.

* Note.—In some localities the standard perch is 18 ft. $\times$ 1 ft. $\times$ $1\frac{1}{2}$ ft. $= 27$ c. ft.

Examples.

1. How many cubic feet are there in a stone block which is 6 ft. 8 in. long, 5 ft. 6 in. broad, and 4 ft. 4 in. thick?

We see that if we were to multiply the first and third of these dimensions, each by 3, we should get for results feet only without odd inches: the second dimension will also be free from odd inches by multiplying it by 2. We shall therefore for simplicity of work use these multiples of the given dimensions, and then compensate for the changes by dividing the final product by $3 \times 3 \times 2 = 18$: the work will then be as here annexed. The result shows that the cubic contents of the block are 159 cubic feet, within $\frac{1}{9}$th of a cubic foot.

$$\begin{array}{r} \text{feet.} \\ 11 \\ 20 \\ \hline 220 \\ 13 \\ \hline 2)2860 \\ \hline 9)1430 \\ \hline 158\frac{8}{9} \text{ cubic ft.} \\ \hline \end{array}$$

By using suitable multiples of the given dimensions computations of this kind may often be abridged in a similar manner; but even here, the greatest possible reduction of work has not been effected: for since the second of the given dimensions becomes freed from odd inches, by whatever even number we multiply it, and because 3 times the first dimension are an even number of feet, namely, 20, our purpose will be accomplished by multiplying the other two dimensions, each by 3, leaving the second dimension as it is, and then dividing the result by 9. In this way the work stands as in the margin. Expedients for abridging the computation, in any such case, are not to be sought for by any long study or examination of the given dimensions, but only when they suggest themselves intuitively upon a slight inspection of those dimensions.

$$\begin{array}{r} \text{feet.} \\ 5\frac{1}{2} \\ 20 \\ \hline 110 \\ 13 \\ \hline 9)1430 \\ \hline 158\frac{8}{9} \\ \hline \end{array}$$

2. How many standard perches of stone-masonry are there in a wall 50 ft. long, 10 ft. 3 in. high, and 1 ft. 4 in. thick?

Using these dimensions as they are, we shall have to divide the product of them by $24\frac{3}{4}$; but by multiplying the second dimension by 4, and this divisor also by 4, we get rid, at once, of the fraction in this latter, and also of the odd inches in 10 ft. 3 in., that is, the second dimension becomes 41 ft., and the divisor 99, and the work is reduced to that in the margin. It is readily seen that the fraction $\frac{60\frac{1}{3}}{99}$ is $\frac{181}{297} = \frac{2}{3}$ nearly: hence there are $27\frac{2}{3}$ standard perches in the wall, nearly. Instead of multiplying by $1\frac{1}{3}$, as here, we might have used the number 4, which is 3 times as great; and then have divided by 297 instead of by 99.

$$\begin{array}{r} \text{feet.} \\ 41 \\ 50 \\ \hline 2050 \\ 1\frac{1}{3} \\ \hline 2050 \\ 683\frac{1}{3} \\ \hline 99)\,2733\frac{1}{3}\,(27\frac{181}{297} \\ 198 \\ \hline 753 \\ 693 \\ \hline 60\frac{1}{3} \\ \hline \end{array}$$

The computer will find it useful, for the purpose of shortening his figure-work, always to recollect that he is at liberty to take the double, treble, &c., of any dimension, provided he take the half, third, &c., of one of the other dimensions; or, leaving both these *unchanged,* provided he also take the double, treble,

&c., of the divisor. He may also take the half, third, &c., of either dimension, provided he take the same fraction, half, third, &c., of the divisor. The following is an illustrative example.

3. How many cubic yards of stone-masonry are there in a wall 114 ft. long, 10 ft. high, and 18 in. thick?

Here we foresee that we shall have to divide the product of these dimensions by 27,—the number of cubic feet in a yard: we shall therefore first double the 18 in., regarding it as 3 feet, and then halve the 10 feet, regarding it as 5 ft. We shall then omit the multiplication by the 3 feet altogether, and use for divisor only the third part of 27, namely, 9; as in the annexed calculation, showing the cubic contents of the masonry to be $63\frac{1}{3}$ yards. We may observe that the above changes in the given dimensions are suggested by simply glancing at them.

$$\begin{array}{r} \text{feet.} \\ 114 \\ 5 \\ \hline 9)\,570 \\ \hline 63\frac{1}{3}\ \text{yds.} \\ \hline \end{array}$$

4. In a chimney-piece the lengths of the mantel and slab are each 4 ft. 6 in., the breadth of both together, 3 ft. 2 in.; the length of each jamb 4 ft. 4 in., and the breadth of both together, 1 ft. 9 in. Required the superficial contents of the whole?

$$\begin{array}{rr} \text{ft.} & \text{in.} \\ 4 & 6 \\ 3 & 2 \\ \hline 13 & 6 \\ & 9 \\ \hline 14\ \text{Ft.} & 3' \\ \hline \end{array} \quad \text{added to} \quad \begin{array}{rr} \text{ft.} & \text{in.} \\ 4 & 4 \\ 1 & 9 \\ \hline 4 & 4 \\ 3 & 3 \\ \hline 7\ \text{Ft.} & 7' \\ \hline \end{array} = 21\tfrac{5}{6}\ \text{sq. ft.}\ \textit{Ans.}$$

IV.—Plasterers', Painters', and Paviors' Work.

Plasterers' work is estimated either by the square yard, or by the square of 100 square feet. Cornices and mouldings, put on after the common plastering is finished, are generally estimated by the linear foot.

Painters compute their work by the square yard, and every part is measured which the brush passes over, the measuring tape being pressed into all cavities and closely applied to all mouldings.

Paviors' work is usually estimated by the square yard.

Examples.

1. The compass of a room is 69 ft. 4 in., and the height of it 10 ft. 3 in.: what will be the expense of plastering the walls at 9*d*. per square yard?

The calculation annexed shows that the number of square feet of plastering is 710 sq. ft. and $\frac{8}{12}$ of a sq. ft; that is, it is $710\frac{2}{3}$ sq. ft. To bring this into sq. yds. we should have to divide it by 9; but since, at 9*d.* per sq. yd., we should have afterwards to multiply by 9, we may dispense with both operations, and regard the result just arrived at as $710\frac{2}{3}$ pence, which is 59*s.* $2\frac{2}{3}$*d.* = £2 19*s.* $2\frac{2}{3}$*d.*

ft.	in.
69	4
10	3
693	4
17	4
710F.	8'

2. A room whose height, girt over with mouldings, is 16 ft. 6 in., is 97 ft. 9 in. in compass: how many sq. yds. of painters' work will it require?

We shall here take twice the first dimension and 4 times the second, and then divide the resulting sq. ft. by 9 × 8 = 72. The number of sq. yds. is thus found to be $179\frac{15}{72} = 179\frac{5}{24}$, or $179\frac{1}{5}$ Yards full.

```
   feet.
   391
    33
  ----
  1173
 1173
72)12903(179 5/24
   72
   --
   570
   504
   ---
    663
    648
    ---
     15
```

3. Required the quantity of plastering and ceiling in a room 14 feet 5 in. by 13 ft. 2 in., the height, up to the under side of the cornice, being 9 ft. 3 in. The cornice girts $8\frac{1}{2}$ in., and projects 5 in. from the wall, at the upper part, next the ceiling. The deductions for apertures are as follows:—one door, 7 ft. by 4 ft., two windows, each 5 ft by $3\frac{1}{2}$ ft., and a fireplace, $5\frac{1}{2}$ ft. by 4 ft.

1st. *For the walls.*

```
                                  ft.  in.
             Length + breadth     27   7
                       Height      9   3
                                 -------
                                 248   3
      Deductions.                  6  10   9
Two windows 5  × 7 = 35          ------------
One door    7  × 4 = 28          255   1   9
Fireplace   5½ × 4 = 22                    2
                     --          ------------
                     85          510   3   6
                     --           85   0   0  (Subtract.)
                                 ------------
                              9) 425   3   6
                                 ---
       Sq. yds. of plastering    47 Yds. 2¼ Ft. 6 In.
```

[The 255 Ft. 1 9 are multiplied by 2, because this is the *measure* of only one side-wall and one end-wall.]

2nd. *For the ceiling.*

	ft.	in.	
14 ft. 5 in. minus 10 in. =	13	7	
13 ft. 2 in. minus 10 in. =	12	4	
	163	0	
	4	6	4
	9) 167	6	4

Sq. yds. of ceiling 18 Yds. 5½ Ft. 4 In.

3rd. *For the cornice.*

	ft.	in.			ft.	in.	
	27	7			27	7	
	0	8½		Or thus:	1	5	
	19	5½	11½		27	7	
			2		11	5	11
Sq. ft. of cornice	39	0	11		39	0	11

Hence the superficial measure of the four lengths of cornice is 39 sq. ft. 11 sq. in. In the second method of computing this we have taken twice the 8½ inches, in anticipation of the final multiplication by 2, and have thus rendered the computation a little easier.

Suppose the common plastering of the walls (rendering, as it is called) cost 10*d.* per sq. yd.; the ceiling 1*s.* 8*d.*; and the cornices 1*s.* 4*d.* per sq. ft.; the account for the above would be as follows.

	£	*s.*	*d.*
Rendering 47¼ yds., at 10*d.*	1	19	4½
Ceiling 18$\frac{11}{18}$ yds., „ 20*d.*	1	11	0¼
Cornice 39$\frac{1}{12}$ Ft., „ 16*d.*	2	12	1¼
Total cost	£6	2*s.*	6*d.*

The foregoing worked examples sufficiently exhibit the methods of computation, whether the work be plastering, painting, or paving; so that the reader can have no difficulty in calculating the following for himself.

4. What will be the charge for plastering a partition 7 ft. 8 in. by 10 ft. 3 in., at 5*d.* per sq. yd., deducting for a door 6 ft. 3 in. by 2 ft. 10 in.? *Ans.* 2*s.* 9¾*d.*

5. What would be the cost of painting and graining four doors on both sides, of which the dimensions of each are 7 ft. 4½ in. by 3 ft. 8¾ in.—the charge for the work being 1*s.* 8*d.* per sq. yd.? *Ans.* £2 0*s.* 9*d.*

6. What will be the expense of paving a rectangular court-yard, which measures 62 ft. 7 in. by 44 ft. 5 in., and through which there is to be laid a footpath along the whole length of it, $5\frac{1}{2}$ ft. wide: the footpath to be paved with flagstones, at 3*s*. per sq. yd., and the rest with pebbles, at 2*s*. 6*d*. per sq. yd.?
Ans. £39 11*s*. $3\frac{1}{4}$*d*.

As the computation of the number of square feet, square yards, &c., in any surface, must be just the same, whether that surface be of plaster, paint, glass, or paper, &c., it would be superfluous, after what has now been done, to extend these articles to the special consideration of window-glazing or room-papering: it will be sufficient merely to mention the following particulars in reference to these.

Glazing.—The dimensions are taken in feet and inches, and the work is computed in square feet; the dimensions for a window are the entire length and width, the cross-bars, separating the panes, being included in the measurement; if the window be circular, or oval, the dimensions are taken just as if it were square, or rectangular; that is, the greatest length and breadth are regarded as the dimensions, for although the quantity of glass actually used, in glazing such a window, is less than the computation assigns to it, yet the extra waste and trouble justify the extra charge.

Room-papering is generally estimated by the square of 100 superficial feet. It is purchased, and laid on, in slips, commonly of 21 inches wide; a complete slip, or piece, is 12 yards long. The length of this paper, for 1 square yard, is $5\frac{1}{7}$ ft., because $5\frac{1}{7} \times 21 = 108$; and as the 21 here is the number of inches and not of feet, we must divide the 108 by 12, to get the number of square feet; the result being 9 sq. ft. = 1 sq. yd. And from thus knowing the length of paper for 9 square feet, we can readily find the length for 100 square feet (a square) thus; $9 : 100 :: 5\frac{1}{7}$ ft. : $57\frac{1}{7}$ ft. = $19\frac{1}{21}$ yds.; so that one dozen (12 yards) and 7 yards of paper, 21 inches wide, will very nearly cover a square; the deficiency being only $\frac{1}{7}$ of a foot, or less than an inch and three-quarters length of paper.

In a similar manner it will be found that a dozen of paper, 21 inches wide, will cover exactly a space of 7 sq. yds.

It may be worth while to remind the reader here that,

in actually working out the proportions just mentioned, and indeed, in working out any proportion, the first and third terms, as also the first and second, may always be multiplied or divided by any number we please, without the result being affected; and that by taking advantage of this circumstance, any fraction in the stating may always be removed: thus, multiplying the first and third terms, in the stating above, by 7, it is converted into the more convenient form:

$$63 : 100 :: 36 \text{ ft.} : 57\tfrac{1}{7} \text{ ft.} = 19\tfrac{1}{21} \text{ yds.}$$

So likewise in the second proportion alluded to for finding the space which a single dozen of paper will cover, the stating $5\frac{1}{7}$ ft. : 36 ft. (12 yds.) :: 1 Yd. : 7 Yds., becomes $36 : 36 \times 7$ :: 1 Yd. : 7 Yds. Or, rather, dividing by the 36, 1 : 7 :: 1 Yd. : 7 Yds.

And these changes, suggested by the above-mentioned general principle, may be at once made, without writing down the original numbers (as here) at all.

We may add, however, that the last result above (7 Yds.) may be readily deduced independently of proportion; for the number of square yards in a piece 12 yards long and 21 inches wide is 21 times 12, divided first by 12, to bring the inches to feet, and then by 3, to bring these feet to yards; and since the division by 12 neutralizes the multiplication by 12, we have only to divide the 21 by 3, giving for result 7 Yds. And in reference to the first proportion, 63 and 36, by the principle before named, might be replaced by 7 and 4, the work then being simply that here annexed; and by thus rejecting factors, common to a multiplier and divisor, figure-work may always be saved.

$$\begin{array}{r} 7)\ 400 \\ \hline 57\tfrac{1}{7} \\ \hline \end{array}$$

Calculation of Wages.

Problem 1.

Knowing the daily wages, or pay, to find the yearly salary.

Rule I. Regard the pence as so many pounds; add to this sum the half of it and also five days' wages.

EXAMPLES.

1. What will 15¾*d.* per day amount to in a year?

	£	*s.*	*d.*
£15¾ =	15	15	0
The half =	7	17	6
Pay for 5 days =	0	6	6¾
	£23	19*s.*	0¾*d.* *Ans.*

By taking the pence for pounds, we virtually multiply the daily pay by 240; that is, we get the pay for 240 days; the half of this is the pay for 120 days; and therefore the two added together, with 5 days' pay besides, gives the pay for 365 days.

RULE II.—Multiply £1 10*s.* 5*d.* by the number of pence per day: the product will be the amount in 1 year; because £1 10*s.* 5*d.* is equal to 365 pence. [This is a very convenient Rule when the daily pay is free from odd farthings.]

2. What will 5*d.* per day amount to in a year?

By Rule I.

£	*s.*	*d.*
5	0	0
2	10	0
	2	1
Ans. £7	12*s.*	1*d.*

By Rule II.

£	*s.*	*d.*
1	10	5
		5
£7	12*s.*	1*d.*

3. What will 18¼*d.* per day amount to in a year? *Ans.* £27 15*s.* 1¼*d.*
4. What will 16*d.* per day amount to in a year? *Ans.* £24 6*s.* 8*d.*
5. What will 3½*d.* per day amount to in a year? *Ans.* £5 6*s.* 5½*d.*

NOTE.—When the pay is only for the ordinary working days, that is, when it is stopped for the 52 Sundays, for Good Friday, and for Christmas Day, only 311 days are paid for in a year. The amount, at 1*d.* a day, is therefore £1 5*s.* 11*d.*; and, at a farthing a day, it is 6*s.* 5¾*d.* Consequently, the Rule will then be this: Multiply £1 5*s.* 11*d.* by the number of pence per day, and 6*s.* 5¾*d.* by the number of additional farthings, and add the results; thus:—

6. What will be the amount of 3¾*d.* per day for 311 days?

	£	*s.*	*d.*	
	1	5	11	
			4	
	5	3	8	
Subtract		6	5¾	for 311 days at ¼*d.*
	£4	17*s.*	2¼*d.*	*Ans.*

Or thus:

	£	s.	d.
	1	5	11
			3
	3	17	9
Add		19	$5\frac{1}{4}$ = 3 times 6s. $5\frac{3}{4}$d.
	£4	17s.	$2\frac{1}{4}$d. *Ans.*

7. At 14*d*. per day, what will be the amount in 311 days? *Ans.* £18 2*s*. 10*d*.
8. If a workman's wages be 5*s*. 10*d*. per day, what is his yearly income? *Ans.* £90 14*s*. 2*d*.

Problem 2. (Converse of Prob. 1.)

Knowing the yearly salary, to find the pay per day.

Rule.—Regard the pounds in the salary as so many pence, and consider the shillings, when 10*s*. or more, as $\frac{1}{2}$*d*., but when less than 10*s*., reject them. Multiply these pence by 2, and divide the product by 3, observing to allow $\frac{1}{4}$*d*. should there be 1 for remainder, and $\frac{3}{4}$*d*. if the remainder be 2; the result will be the daily income nearly, if not to the nearest farthing.

Note.—The year is here the whole 365 days.

Examples.

1. If a servant's wages be £24 a year, what is his daily pay?
   24*d*. $\times$ 2 $\div$ 3 = 8*d*. $\times$ 2 = 16*d*., to the nearest farthing.
2. If the yearly wages be 26 guineas, what is the pay per day?
   27*d*. $\times$ 2 $\div$ 3 = 9*d*. $\times$ 2 = 18*d*., to the nearest farthing.
3. If the yearly wages be £23 15*s*., what is the pay per day?
   $23\frac{1}{2}$*d*. $\times$ 2 $\div$ 3 = 47*d*. $\div$ 3 = $15\frac{3}{4}$*d*., to the nearest farthing.
4. If the yearly income be £150, what is the amount per day?
   150*d*. $\times$ 2 $\div$ 3 = 100*d*. = 8*s*. 4*d*. nearly.

The foregoing Rule is suggested by the following considerations. If the year consisted of only 360 days instead of 365, then, by dividing the salary by 360, we should get the daily pay. Now if the salary expressed in pounds be regarded as so many pence, it becomes virtually divided by $20 \times 12 = 240$; so that the salary is 240 times this number of pence; but multiplying anything by 240, and then dividing by 360, is the same as multiplying by 2, and then dividing by 3; and hence by proceeding in this way, the 360th part of the salary will be accurately determined. And since the

difference between the 360th part and the 365th part of a comparatively small sum is but trifling—being only the $\frac{1}{360 \times 73}$ part of the whole, which is but a farthing in £28 —it follows that, for an income not exceeding £27, the error cannot be so great as a farthing; and it is always in excess. If a few shillings—say 5*s.*—be deducted from the yearly income, this excess will be pretty nearly counterbalanced; for then there will be deducted from the daily pay $\frac{5}{365}$*s.*, about ⅔ of a farthing. For an income much above the limit (£28) this deduction will be insufficient; and to avoid division into cases, and likewise to preclude a fractional remainder, from the divisor 3, the Rule recommends, generally, that if the shillings connected with the pounds, in the income, be fewer than 10*s.*, they should be rejected, as also the overplus above 10*s.* As the Rule is professedly only (in most cases) a close approximation to the strict truth, and as, from its simplicity, it is so easily worked mentally, it is retained in this edition. We merely caution the reader that it is not to be relied upon, within a halfpenny or penny of the truth, for incomes yielding several shillings per day. Thus, take the yearly income at £150; the Table below shows that, to the nearest farthing, the daily pay is 8*s.* 2¾*d.*; we have seen above (Ex. 4) that the Rule makes it 8*s.* 4*d.*, which is 1¼*d.* too much. On the whole, when the daily payment is to be found with greater exactness, we recommend recourse to the subjoined Table.

WAGES.

*Table of salaries, etc., from £1 to £150 per annum, reduced to so much per month, per week, per day.**

Y.	Pr. M.	Pr. W.	Pr. D.	Y.	Pr. M.	Pr. W.	Pr. D.	Y.	Pr. M.	Pr. W.	Pr. D.
£	s. d.	s. d.	s. d.	£	£ s. d.	s. d.	s. d.	£	£ s. d.	£ s. d.	s. d.
1	1 8	0 4¾	0 0¾	11	0 18 4	4 2¾	0 7¼	30	2 10 0	0 11 6½	1 7¾
2	3 4	0 9¼	0 1¼	12	1 0 0	4 7¼	0 8	40	3 6 8	0 15 4½	2 2¼
3	5 0	1 1¾	0 2	13	1 1 8	5 0	0 8½	50	4 3 4	0 19 2¾	2 9
4	6 8	1 6½	0 2¾	14	1 3 4	5 4½	0 9¼	60	5 0 0	1 3 0¾	3 3½
5	8 4	1 11	0 3¼	15	1 5 0	5 9¼	0 10	70	5 16 8	1 6 11	3 10
6	10 0	2 3¾	0 4	16	1 6 8	6 1¾	0 10½	80	6 13 4	1 10 9¼	4 4½
7	11 8	2 8¼	0 4½	17	1 8 4	6 6½	0 11¼	90	7 10 0	1 14 7½	4 11¼
8	13 4	3 0¾	0 5¼	18	1 10 0	6 11	0 11¾	100	8 6 8	1 18 5½	5 5¾
9	15 0	3 5½	0 6	19	1 11 8	7 3¼	1 0½	125	10 8 4	2 8 0	6 10¼
10	16 8	3 10	0 6½	20	1 13 4	7 8¼	1 1¼	150	12 10 0	2 17 8¼	8 2¾

NOTE.—One farthing per day is 7*s.* 7¼*d.* per year.

* *The above* table is calculated to the nearest amount that either *employer or* employed can insist upon.

We shall now show the use of this Table by applying it to the Examples worked by the Rule (p. 171).

	Ex. 1. per day.		Ex. 2.		Ex. 3.
For £20	1*s*. 1¼*d*.	For £20	1*s*. 1¼*d*.	For £20	1*s*. 1¼*d*.
,, £ 4	2¾	,, £ 7	4½	,, £ 3	2
		6*s*. (Note)	¼	15*s*. (Note)	½
,, £24	1*s*. 4*d*.		1*s*. 6*d*.		1*s*. 3¾*d*.

We thus see that for such small incomes as these, the Rule is sufficiently accurate; but, as already shown, for an income of £150 it errs in excess by 1¼*d*.

As another Example, let us take an income of £40.

By the Rule.
3) 80*d*.
26¾*d*. = 2*s*. 2¾*d*.

By the Table.
For £40 2*s*. 2¼*d*.

The difference here is ½*d*.

If the income had been £100, the difference would have been one penny.

In concluding these remarks, it may be as well to observe that the reason why the Rule directs that when the remainder, from the division by 3, is 1, a farthing should be allowed for it, but that when it is 2, three farthings should be allowed, is because ⅓*d*. = 4/3*f*. = 1*f*. *and* ⅓*f*.; and ⅔*d*. = 8/3*f*. = 3*f*. all but ⅓*f*.

Problem 3.

The number of shillings in a week's earnings being given, to find the earnings per year.

Rule.—Add together 2½ times as many pounds as there are shillings, and twice the shillings themselves: the result will be the earnings for the year.

For 20 times the number of shillings make so many pounds, and these pounds are the earnings of 20 weeks; so that 2½ times this sum must be the earnings of 50 weeks; and twice one week's earnings being added, the result must be the earnings in a year.

Examples.

1. A year's earnings, at 11*s*. a week, are £11 × 2½ + 22*s*. = £28 12*s*.
2. A year's earnings, at 16*s*. a week, are £16 × 2½ + 32*s*. = £41 12*s*.

3. A year's earnings, at 18*s*. 6*d*. a week, are £18½ × 2½ + 37*s*. = £37 + £9 5*s*. + £1 17*s*. = £48 2*s*. Or thus: Since 52*d*. = 4*s*. 4*d*., we have only to add six times this, namely, £1 6*s*., to the year's income at 18*s*. per week: so that the work may stand as follows, namely, £18 × 2½ + £1 16*s*. + £1 6*s*. = £45 + £3 2*s*. = £48 2*s*. And in this manner we may always proceed when there are odd pence in the week's wages.
4. How much is earned in a year, at the rate of 16*s* 10*d*. per week? *Ans*. £43 15*s*. 4*d*.
5. If a family spend, on the average, 7*s*. 3½*d*. per week for bread, what is the expenditure per year? *Ans*. £18 19*s*. 2*d*.
6. What is the yearly income of a person who earns £2 17*s*. 8¼*d*. per week? *Ans*. £149 19*s*. 9*d*.

[See the Table, in which the weekly pay, for £150 a year, is stated to be the above sum, and allowably so; because the overplus 3*d*., divided by 52, gives a fraction too small for representation as money.]

Calculation of Interest.

Problem 1.

To find the interest of any sum of money, at five per cent. per annum.

Rule.—Regard the pounds as so many shillings, and allow at the rate of threepence for every additional 5*s*.; and, consequently, one farthing for every 5*d*. The interest, at 5 per cent., will then be expressed.

Examples.

1. The interest, for one year, on £75, at 5 per cent., is 75*s*. = £3 15*s*.
2. The interest, for one year, on £110, at 5 per cent., is 110*s*. = £5 10*s*.
3. The interest, for one year, on £69 15*s*., at 5 per cent., is 69*s*. 9*d*. = £3 9*s*. 9*d*.
4. What is the interest on £587 16*s*. 4*d*., for 7 years at 5 per cent.?

[The interest of a sum for 7 years is, of course, the interest of 7 times that sum for one year.]

	£	s.	d.				d.	
	587	16	4					
			7					
				Interest for		15*s*. =	9	
2,0)	411,4	14	4	,,	,,	10*d*. =	½	Subtract.
	£205	14*s*.	8½*d*.	,, for		14*s*. 2*d*. =	8½*d*.	

Here £4114 are regarded as so many shillings, and are therefore divided by 20, to bring them into pounds. The interest for 14*s.* 2*d.* is then found separately, according to the Rule: the interest for the additional 2*d.*, being less than half a farthing, is disregarded. Of course the work for the 14*s.* 2*d.* may be readily executed mentally, and the resulting 8½*d.* annexed at once to the £205 14*s.*

5. What is the interest on £26 5*s.*, for 1 year, at 5 per cent.? *Ans.* £1 6*s.* 3*d.*
6. What is the interest on £47 10*s.*, for 1 year, at 5 per cent.? *Ans.* £2 7*s.* 6*d.*
7. What is the interest on £9826 13*s.* 8*d.*, for 1 year, at 5 per cent.? *Ans.* £491 6*s.* 8*d.*

Problem 2.

To find the interest at any rate per cent. per annum.

Rule I.—Find the interest at 5 per cent. by the last problem: one-fifth of this will be the interest at 1 per cent.; and this, multiplied by the given rate, will give the interest at that rate. But for certain rates the calculation may be abbreviated by taking parts, as in the third of the following Examples.

Examples.

1. What is the interest on £587 16*s.* 4*d.*, for 7 years, at 3 per cent., at 4 per cent., and at 6 per cent.?

	£	*s.*	*d.*	
	587	16	4	
			7 years	
2,0)	411,4	14	4	
5)	205	14	8½	, 5 per cent. (*See last page.*)
3 ×	41	2	11¼	, 1 per cent.
=	123	8	9¾	, 3 per cent.

£	*s.*	*d.*	
205	14	8½	, 5 per cent.
41	2	11¼	, 1 "
164	11	9¼	, 4 "
246	17	7¾	, 6 "

In the calculation for 4 and 6 per cent., the interest at 1 per cent. is subtracted for the former rate, and added for the latter rate.

Note.—It is generally preferable to multiply by the number of years first, as above, since then there will never be any fraction of a penny to multiply.

2. What is the interest on £212 10*s*. 4*d*., for $2\frac{3}{4}$ years, at $2\frac{1}{2}$ per cent.?

	£	*s*.	*d*.	
4)	212	10	4	
			3 years.	[$2\frac{3}{4} = 3 - \frac{1}{4}$.]
	637	11	0	
Subtract	53	2	7	
2,0)	58,4	8	5	
2)	29	4	5	= Interest at 5 per cent.
Ans.	£14	12*s*.	$2\frac{1}{2}$*d*.	= „ $2\frac{1}{2}$ „

3. What is the interest on £500, for 4 years, at £5 7*s*. 6*d*. per cent?

Instead of multiplying the 500 by 4, and then dividing by 20, we shall divide simply by 5, rejecting the factor 4 from both multiplier and divisor.

	£		
	100	= Interest for 4 years, at	5 per cent.
20th part of £100	= 5	„	5*s*. „
$\frac{1}{2}$ of the above	= 2 10*s*.	„	2*s*. 6*d*. „
Ans.	£107 10*s*.	„	£5 7*s*. 6*d*. „

4. What is the interest on £896, for $2\frac{1}{2}$ years, at $3\frac{1}{4}$ per cent.?

	£	
	896	
	$2\frac{1}{2}$ years.	
	1792	
	448	
2,0)	224,0	
5)	112 £	= Int. at 5 per cent.
	£22 8*s*.	„ 1 „
	$3\frac{1}{4}$ the rate	
	67 4	
For the $\frac{1}{4}$	5 12	
Ans.	£72 16*s*.	Int. at $3\frac{1}{4}$ „

Or thus:

	£
	896
	$2\frac{1}{2}$
	1792
	448
4)	2240
	$3\frac{1}{4}$
	6720
	560
	£72,80
	20
	16,00*s*.

The first of these methods admits of being considerably shortened, in consequence of the number of years being $2\frac{1}{2}$. If we take double this number, there will be first a multiplication by 5, and afterwards a division by 5,—two neutralizing operations. We may therefore suppress both; so that we shall get the interest, at 1 per cent. by simply

dividing £896 by the double of 20, seeing that $2\frac{1}{2}$ has been doubled; that is, the work for the 1 per cent. will be merely this:

$$£896 \div 40 = £22\tfrac{2}{5} = £22\ 8s.$$

In the second method, instead of dividing by 20 and 5, and then multiplying by $3\frac{1}{4}$, we have first multiplied by $3\frac{1}{4}$, and have then divided by 100, which is the same as dividing by 20 and 5 in succession. This second method is in accordance with the following general Rule.

RULE II.—Multiply the principal by the number of years, the product by the rate, and divide the result by 100. Or multiply the principal by the product of the rate and number of years, and divide the result by 100.

NOTE.—1. If the product of the rate and number of years should be 10 (as in Ex. 2 following), then instead of multiplying by this 10, and afterwards dividing by 100, we should omit the multiplication altogether, and merely divide the principal by 10.

2. And if the product of the rate and number of years should be 20 (as in Ex. 4), then the multiplication by the 20 should, in like manner, be omitted, and the principal be divided by 5 only. In the former case, the factor 10 is suppressed in both multiplier and divisor, and in the latter case, the factor 20.

It should ever be present to the mind of the computer that, in any calculation whatever, factors common to a multiplier and a divisor may always be discarded, however far apart these two operations of multiplication and division may be, provided that no operations, except those of multiplication and division, intervene.

EXAMPLES.

1. What is the interest of £587 16*s.* 4*d.*, at 6 per cent., for 7 years? (Ex. 1, p. 175.)

$$\begin{array}{rrrl}
£ & s. & d. & \\
587 & 16 & 4 & \\
 & & 7 & \text{years.} \\
\hline
4114 & 14 & 4 & \\
 & & 6, & \text{the rate.} \\
\hline
246{,}88 & 6 & 0 & \\
20 & & & \\
\hline
17{,}66 & & & \\
12 & & & \\
\hline
7{,}92 & & & \textit{Ans. } £246\ 17s.\ 7\tfrac{3}{4}d.\text{; or more} \\
4 & & & \quad\text{nearly, } £246\ 17s.\ 8d. \\
\hline
3{,}68 & & &
\end{array}$$

Whether this Rule or the former one be the more eligible, in any particular case, must be left to the computer to determine. It may be well for the reader to work the following examples by both Rules, keeping in remembrance, however, the foregoing Note.

2. What is the interest on £765 12*s.* 7*d.*, for 4 years, at $2\frac{1}{2}$ per cent.? *Ans.* £76 11*s.* 3*d.*
3. What is the interest on £78 12*s.* 10*d.*, for $12\frac{1}{2}$ years, at 1 per cent.? *Ans.* £9 16*s.* $7\frac{1}{4}$*d.*
4. What is the interest on £325 7*s.* 6*d.*, for $3\frac{1}{3}$ years, at 6 per cent.? *Ans.* £65 1*s.* 6*d.*
5. What is the interest on £193 12*s.*, for 1 year, at £11 18*s.* 6*d.* per cent.? *Ans.* £22 2*s.* $4\frac{1}{2}$*d.*

The following short Table may be found useful on many occasions.

Table of the Interest on £1 *for* 1 *year, at from* 1 *to* 10 *per cent.*

Per cent.	Interest. *s.*	*d.*	Per cent.	Interest. *s.*	*d.*
1 . . .	0	$2\frac{2}{5}$	6 . . .	1	$2\frac{2}{5}$
2 . . .	0	$4\frac{4}{5}$	7 . . .	1	$4\frac{4}{5}$
3 . . .	0	$7\frac{1}{5}$	8 . . .	1	$7\frac{1}{5}$
4 . . .	0	$9\frac{3}{5}$	9 . . .	1	$9\frac{3}{5}$
5 . . .	1	0	10 . . .	2	0

If it be only borne in mind that the interest of £1 for 1 year, at 1 per cent., is $2\frac{2}{5}d.$; or without any effort of memory, if we reduce mentally $\frac{240}{100}d.$, that is, $\frac{12}{5}d.$, to $2\frac{2}{5}d.$, the interest for 1 year, at any other rate per cent., will be obtained by simply multiplying $2\frac{2}{5}d.$ by that rate: thus, $2\frac{2}{5}d. \times 3 = 7\frac{1}{5}d.$, the interest of £1, for 1 year, at 3 per cent.; also $2\frac{2}{5}d. \times 7 = 1s.\ 4\frac{4}{5}d.$, the interest of £1, for 1 year, at 7 per cent.; and so on.

By way of application, let us take Ex. 4, p. 176, the working of which will be as follows, the interest being calculated first for 3 per cent., and then that for $\frac{1}{5}$ per cent. added.

896 No of £'s.
$2\frac{1}{2}$,, years.
————
1792
448
————
2240
(*Number from the Table.*) $7\frac{1}{5}d.$
————
15680
448
————
12) 16128*d.*
————
2,0) 134,4*s.*
————
Interest for $2\frac{1}{2}$ years, at 3 per cent. £67 4*s.*

But we may work in a preferable manner, thus:—$2\frac{1}{2} \times 3 = 7\frac{1}{2}$; therefore the interest is the same as that for 1 year at $7\frac{1}{2}$ per cent.

The interest of £896, at	5 per cent	= £44	16*s.*
"	$2\frac{1}{2}$ "	= 22	8
"	$7\frac{1}{2}$ "	= £67	4*s.*

And we thus see, in particular cases, that a general Rule may sometimes be superseded by a shorter method.

If one twelfth of this be added to it, the result will be £67 4*s.* + £5 12*s.* = £72 16*s.*, the interest at $3\frac{1}{4}$ per cent., for $2\frac{1}{2}$ years.

PROBLEM 3.

To find the interest on any number of pounds, at any rate per cent., for any number of months.

GENERAL RULE.—Regard the pounds as so many pence; multiply this number, the number of months, and the double of the rate together; a tenth part of the product will be the interest in pence.

[We recommend taking twice the rate, and dividing by 10, rather than taking the rate itself, and dividing by 5, for two reasons: first, by doubling the rate, we get rid of the fraction $\frac{1}{2}$, should it enter the rate: and, secondly, twice the rate can be written down quite as readily as the rate itself; and, easy as division by 5 may be, division by 10 is still easier; in fact division by 10 involves no actual figure-work at all. Of course, if seen to be the more convenient, we may divide either of the three factors to be multiplied together by 10 at the outset, instead of delaying the division till their product is obtained; and it always will be the more convenient to do this when either of the factors terminates with a cipher, as in Ex. 2, next page.]

EXAMPLES.

1. What is the interest on £36, for 3 months, at $2\frac{1}{2}$ per cent.?
   36*d.* $\times 3 \times 5 = 540$: hence the interest is 54*d.* = 4*s.* 6*d.* *Ans.*
   Or thus: 3*s.* $\times 3 \times 5 =$ 45*s.*; and 45*s.* $\div 10 =$ 4*s.* 6*d.* *Ans.*

The reason of the foregoing Rule will appear from the following considerations.

$\frac{\text{Principal} \times \text{Rate}}{100}$ is the interest for 12 months; therefore

$\frac{1}{12}$ of this is the interest for 1 month: but $\frac{1}{12}$ is the same as $\frac{20}{240}$; so that instead of dividing the interest for 12 months by 12, we may divide it by 240, and then multiply by 20. Now by writing the pounds as so many pence, we do, virtually, divide by 240; and the multiplying by 20, and dividing by 100 is, in effect, the same as expunging the factor 20 in the divisor, 100, thus leaving for divisor only 5, and not multiplying at all; and this divisor, 5, is converted into 10 by multiplying it and the numerator by 2; that is, by taking twice the rate. In this way the interest for 1 month (meaning by a month the 12th part of a year) is the number of pounds in the principal (taken as so many pence) multiplied by twice the rate, and the product divided by 10; and consequently, for any number of months, we have only to introduce that number as an additional multiplier. And this is the Rule.

2. What is the interest on £220, for 11 months, at 3 per cent.?
Dividing by 10 first, 1*s.* 10*d.* $\times$ 6 = 11*s.*; and 11 times this is 121*s.* = £6 1*s.* *Ans.*

3. What is the interest on £245 13*s.* 4*d.*, for 1 month, at 3 per cent.?
13*s.* 4*d.* = £$\frac{2}{3}$; and 245$\frac{2}{3}$*d.* $\times$ 6 is 245 sixpences + 4*d.*; that is, it is 122*s.* 10*d.*, the tenth part of which is 12*s.* 3$\frac{2}{5}$*d.*, the interest required.

4. What is the interest on £144 15*s.*, for 9 months, at 5 per cent.?
15*s.* = £$\frac{3}{4}$; and 144$\frac{3}{4}$*d.* $\times$ 9 $\times$ 10 $\div$ 10 = 12*s.* 0$\frac{3}{4}$*d.* $\times$ 9 = £5 8*s.* 6$\frac{3}{4}$*d.* *Ans.*

[The 12*s.* 0$\frac{3}{4}$*d.* might be written at once from mere inspection of the principal.]

NOTE.—We see by this Example, that when the rate is 5 per cent., the multiplication by double the rate, and the division by 10, being operations which neutralize each other, may both be omitted: it is sufficient to multiply the number of pounds, taken as so many pence, by the number of months. It is proper to observe that when the shillings and pence, connected with the pounds in the principal, do not make a convenient fraction of £1, they may be subdivided into convenient fractions (none of them being more complicated than £$\frac{1}{16}$, without making any error greater than £$\frac{1}{32}$ in the principal; and this extreme error, even when the rate is so great as 10 per cent., and the months so many as 11, can cause an error in the interest equal only to

$$\tfrac{1}{32}d. \times 11 \times 20 \div 10 = \tfrac{1}{32}d. \times 11 \times 2 = \tfrac{22}{32}d., \text{ less than } \tfrac{3}{4}d.$$

For instance: suppose the principal in the last Example above had

been £144 16*s.* 7*d.* Then we might have subdivided the 16*s.* 7*d.* into 15*s.* + 1*s.* 3*d.* + 4*d* = £$\frac{3}{4}$ + £$\frac{1}{16}$ + 4*d.*, and have disregarded the 4*d.*, as having no sensible influence on the resulting interest: the interest, in the Example above, with the foregoing addition of 1*s.* 7*d.* to the principal, would have to be increased by $\frac{1}{16}$*d.* × 9 = $\frac{9}{16}$*d.*, or one half-penny. But regarding the addition to be 1*s.* 8*d.*, the increase of interest would be $\frac{1}{12}$*d.* × 9 = $\frac{3}{4}$*d.*

The general Rule by which the foregoing Examples have been worked, though sufficiently easy, becomes still further simplified for the rates 3 per cent., 5 per cent., and 6 per cent. In these cases it may be expressed in the three forms following.

1. *For 3 per cent.*—Regard the pounds in the principal as so many shillings; multiply these by the number of months, and divide by 20.

It is easy to see how this follows from the General Rule at p. 179. The double rate here is 6; and since the divisor is always 10, if we double the multiplier (6) and double also this 10, the pence in the Rule become virtually so many shillings; and instead of 10, the divisor becomes 20.

2. *For 5 per cent.*—Regard the pounds as so many pence; and multiply these by the number of months. (See Note, p. 180.)

3. *For 6 per cent.*—Regard the pounds as so many shillings; multiply these by the number of months, and divide the product by 10. [This is an obvious inference from the above Rule for 3 per cent.]

5. What is the interest on £87, for 5 months, at 3$\frac{3}{4}$ per cent.?
Ans. £1 7*s.* 2$\frac{1}{4}$*d.*
6. What is the interest on £110, for 9 months, at 5 per cent.?
Ans. £4 2*s.* 6*d.*
7. What is the interest on £90, for 8 months, at 6 per cent.?
Ans. £3 12*s.*
8. What is the interest on £619 9*s.* 6*d.*, for 7 months, at 5$\frac{1}{2}$ per cent.?
Ans. £19 17*s.* 6*d.*

Problem 4.

To find the interest on any principal, at any rate per cent., for any number of days.

The ordinary and obvious Rule for this is—As 365 is to the proposed number of days, so is the interest for 1 year to the interest required. But, since twice 365 is 730, and

since, moreover, in finding the interest in the usual way, for 1 year, we have to divide by 100, after multiplying the principal by the rate, we may obviously proceed as follows.

RULE I.—Multiply the product of the principal and twice the rate by the number of days, and divide the result by 73000.

Now in dividing by so large a number as this, it is plain that a few shillings, more or less, in the dividend, cannot cause any appreciable difference in the result; the 73000th part of so much as 10*s.*, is only the 7300th part of 1*s.*, which is less than the 608th part of a penny. We may therefore safely take the sum, to be divided, to the nearest pound only; increasing the number of pounds by a unit, when 10*s.* and upwards are connected with the pounds, and rejecting the overplus shillings altogether when they amount to less than 10*s.* By so doing, the error in the quotient can never be so great as the 608th part of a penny, as we have just seen.

EXAMPLES.

1. What is the interest on £325 7*s.*, for 89 days, at $4\frac{1}{2}$ per cent.?

£325 7*s.* × 9 = £2928 3*s.*; and this multiplied by 89 gives for product £260605 7*s.*; and rejecting the 7*s.*, as of no moment, the remainder of the work is that here annexed.

```
               £
73,000) 260,605 (3£
        219
        -----
         41605
            20
        -------
        832,100 (11s.
        803
        ----
         291
          12
        -----
        349,2 (4d.
        292
        ----
         572
           4
        -----
        228,8 (3f.
        219
        -----
           9
```

It thus appears that the interest required is £3 11*s.* $4\frac{3}{4}$*d.*

We have spoken above of the insignificant influence, upon the result, of the odd shillings connected with the pounds to be divided by the number 73000; it may be interesting and instructive to the reader to test for himself the trifling effect, upon the foregoing result, which would ensue from increasing the dividend here employed by so much as £120, thus converting it into £260725: he will find that the answer will differ from that arrived at above by less than one halfpenny.

But there is a more expeditious way of arriving at the quotient, in a case of this kind, than by actually dividing by 73000: it is as follows.

RULE II.—Conceive the dividend, that is, the product of *the principal,* the double rate, and the number of days, *to*

be divided by 100000; that is, cut off the last five figures of it, and then divide by 3: divide the quotient by 10, or, which is the same thing, remove the preceding figures each one place further to the right; then divide again by 10, in a similar way: the results being in column, add all up; the sum will be the quotient expressed in pounds.

	£
3)	2·60605
	·86868
	8686
	868
	£3·57027
	20
	11·4054*s.*
	12
	4·8648*d.*
	4
	3·4592*f.*

The work of the foregoing Example, by this Rule, is here annexed: the answer brought out is £3 11*s.* 4$\frac{3}{4}$*d.*, as before; the neglect of the decimals, after the fifth place, not affecting even the farthings.

It may be here noticed, however, that in calculations of Interest, the amount to the nearest penny is, in general, all that is demanded: in the present case the interest charged would be £3 11*s.* 5*d.*

We shall give another worked Example in illustration of the foregoing Rule.

2. What is the interest on £956 14*s.* 6*d.*, for 7 days, at 4$\frac{1}{2}$ per cent?

£	*s.*	*d*	
956	14	6	
		9	double the rate.
8610	10	6	
		7	No. of days.
£60273	13*s.*	6*d.*	

3)	·60274
	·20091
	2009
	200
	£·82574
	20
	16·5148*s.*
	12
	6·1776*d.*

The answer is 16*s.* 6*d.* And similarly in all other cases in which the interest, for the specified number of days, does not exceed £10; or in which accuracy in the fifth place of figures, in the sum of the four numbers added together, is of no moment. We shall see hereafter, p. 185 why accuracy to the nearest farthing cannot be counted upon if the interest exceed £10. As far as this limit, as to the interest, Rule II. may always be safely depended upon; and therefore it cannot but be acceptable to persons engaged in computations of this kind in Savings Banks.

NOTE.—When the interest is 5 per cent., then since, in this case, the double of the rate is 10, we need multiply only by the number of days,

and point off but four places of figures from the result, instead of five; and whenever the dividend happens to be so small as not to have so many figures as it is necessary to point off, we must prefix to it as many ciphers as will suffice to make up the required number of places.

3. What is the interest on £375, for 12 days, at $3\frac{1}{2}$ per cent.?
Ans. 8*s.* 6*d.*
4. What is the interest on £370, for 40 days, at 5 per cent.?
Ans. £2 0*s.* 6*d.*
5. What is the interest on £3204 14*s.*, for 37 days, at 5 per cent.?
Ans. £16 4*s.* 10*d.*
6. What is the interest on £950, for 80 days, at 7 per cent.?
Ans. £14 11*s.* 6*d.*

[In the foregoing Examples the interest is determined to the nearest penny. It may be satisfactory to the reader to work Examples 5 and 6 by both Rules.]

It remains for us now to explain the principle upon which the foregoing easy and expeditious method of computing the interest for a specified number of days is founded.

In every operation of Division, we know that when the complete quotient is obtained, the product of this quotient and the divisor will give the dividend; that is to say, that there will always be the following equality, viz., Quotient × Divisor = Dividend. And further, that by whatever number we divide both divisor and dividend, before actually employing them as such, the quotient will remain just the same.

$$\begin{array}{r} 3)\ \cdot73000 \\ \cdot24333\frac{1}{3} \\ \cdot02433\frac{1}{3} \\ \cdot00243\frac{1}{3} \\ \hline 1\cdot0001 \\ \hline \end{array}$$

NOTE. $\frac{1}{3}$ is $\cdot333\ldots$ continued interminably.

Suppose, in the case before us, that we in this way reduce the divisor 73000, more and more, by successively dividing it by 100000, 3, 10, and 10, as in the annexed operation; the division by the number 100000 reducing the whole number 73000 to the decimal ·73000, the superfluous ciphers here being retained merely for symmetry sake. Then provided we reduce the dividend in exactly the same way, we know, from the general principle stated above, that by employing each reduced divisor, in conjunction with the corresponding reduced dividend, the equality referred to always has place—the quotient continuing unaltered; so that this unaltered quotient, multiplied by any one of the varying divisors, will always produce the dividend in connection with that divisor. Consequently the *fixed and constant* quotient, multiplied by the sum of all the

varying divisors, must give a product equal to the sum of all the varying dividends; in other words, there will be the equality following, which, for the purpose of future reference, we shall mark [A].

Quotient $\times$ 1·0001 = Sum of all the several Dividends . . . . [A].

Now the multiplier 1·0001 differs from 1 by an amount so small that, in calculations such as those in which we are now engaged, the difference is inappreciable, and the number may with safety be replaced by 1 itself; in which case, the left-hand member of the foregoing equality becomes simply—Quotient = Sum of all the Dividends; and hence the quotient arising from the division by 73000, in accordance with Rule I., may be more expeditiously found by summing up all the reduced dividends arrived at conformably to the directions in Rule II.; in all those cases, that is, in which the replacing the multiplier 1·0001, as used above, by 1, can lead to no error of consequence; in other words, whenever the Interest (the Quotient) is not so large a sum as for the ten-thousandth part of it to be appreciable. [The 10000th part of the Quotient (or Interest) is Quotient $\times$ ·0001,—the amount which is rejected by the Rule.]

Now we know that £1 is equal to 960 farthings; consequently, £10 is equal to 9600 farthings; and this number, being less than 10000, the 10000th part of it is less than a farthing; it is, in fact, the decimal ·96 *f.* We may infer therefore, from what is said above, that whenever the required interest is foreseen to be a sum not exceeding £10—and whether 10 times the divisor 73000 exceeds the dividend or not, may be ascertained at a glance,—Rule II. may be depended upon for the accurate determination of that interest, within a farthing; the error being a fraction of a farthing in excess.

NOTE.—The 10000th part of the "Sum of all the Dividends" is not, in strictness, the same as the 10000th part of the Interest, or Quotient:—it is the 10000th part of the Quotient, and the 10000th part of that part besides; as is evident from the equality marked [A] above.

This addition, however, is so utterly insignificant, that Rule II. very properly ignores its existence.

We may here remark, however, that in working Examples by this Rule, the decimals are not extended beyond five places, and therefore that appreciable error may be suspected to arise from this cause: let us see whether or not such can be the case. In the first division of the

proposed dividend, namely, the division by 3, the greatest remainder that can arise is obviously 2, for which remainder the continuation of the decimals would be the figures 666... Now even if the figures following the fifth place, in the two subsequent divisions (by 10), could be a row of 9's, the sum of the column of decimals, immediately beyond the fifth place, would amount only to the number 26; so that the error arising from rejecting these decimals would be more than compensated by increasing the fifth decimal by 3; and the value of this increase in the pounds would be only $960 \times \cdot00003 = \cdot0288$ farthings. In every case, the error from curtailing the decimals is, therefore, an error of a fraction of a farthing in defect. We have already seen that when the Interest is not more than £10, the error of Rule II., which arises from replacing 1·0001 by 1, is likewise an error of only a fraction of a farthing, and that this error is in excess. Hence the Interest, as computed by Rule II., whenever that Interest does not exceed £10, is affected with an error which is merely the difference between two fractions of a farthing, and is therefore inappreciable.

Calculations respecting Commission, Brokerage, Insurance, &c.

Problem.

To find the Commission or Brokerage upon any number of pounds, at a given rate per cent.

Rule I.—Regard the given number of pounds as so many shillings; multiply by the rate per cent., and divide by 5. Or,

Rule II.—Multiply the given number of pounds, taken as so many shillings, by twice the rate per cent., and divide by 10. Or,

Rule III.—Multiply the pounds, taken as so many shillings, by twice the rate per cent., and from the product cut off the unit's figure: the remaining figures express shillings, and the figure cut off denotes so many pence and fifths of a penny.

Examples.

1. What is the commission on £83, at 2 per cent.?

By Rule I.	By Rule II.	By Rule III.
83*s*.	83*s*.	83*s*.
2	4	4
5) 166	1,0) 33,2	33,2 = 33*s*. $2\frac{2}{5}$*d*.
Ans. 33*s*. $2\frac{2}{5}$*d*.	$33\frac{2}{10}$*s*. = 33*s*. $2\frac{2}{5}$*d*.	

2. **What is the commission on £58, at $6\frac{1}{4}$ per cent. ?**

By Rule I.

$$\begin{array}{l} 58s. \\ \underline{\quad 6\tfrac{1}{4}} \\ 348 \\ \underline{\;\,14 \quad 6} \\ 5)\ \underline{362s.\ 6d.} \\ \textit{Ans.}\ 72s.\ 6d. \end{array}$$

By Rule II.

$$\begin{array}{l} 58s. \\ \underline{12\tfrac{1}{2}} \\ 696 \\ \underline{\;\,29} \\ 72{,}5 = 72\tfrac{1}{2}s. = \\ \quad 72s.6d. \end{array}$$

By Rule III.

$$\begin{array}{l} 58s. \\ \underline{12\tfrac{1}{2}} \\ 696 \\ \underline{\;\,29} \\ 72{,}5 = 72s.\ 5\tfrac{5}{8}d. \\ \quad 72s.\ 6d. \end{array}$$

3. **What is the commission on £125, at $3\frac{3}{8}$ per cent. ?**

By Rule I.

$$\begin{array}{r} 125s. \\ \underline{3\tfrac{3}{8}} \\ 375 \\ 375 \div 8 = \underline{46 \quad 10\tfrac{1}{2}} \\ 5)\ \underline{421s.\ 10\tfrac{1}{2}d.} \\ \textit{Ans.}\ 84s.\ \ 4\tfrac{1}{2}d. \end{array}$$

By Rule II.

$$\begin{array}{l} 125s. \\ \underline{\quad 6\tfrac{3}{4}} \\ 750 \\ \underline{\;\,93 \quad 9} \\ 1{,}0)\ \underline{84{,}3s.\ 9d.} \\ \textit{Ans.}\ 84s.\ \ 4\tfrac{1}{2}d. \end{array} \qquad = \qquad \begin{array}{l} 4)\ \underline{375s.} \\ \underline{93s.\ 9d.} = 93\tfrac{3}{4} \end{array}$$

By Rule III.

$$\begin{array}{l} 125s. \\ \underline{\quad 6\tfrac{3}{4}} \\ 750 \\ \underline{\;\,93\tfrac{3}{4}} \end{array}$$

$84{,}3\frac{3}{4} = 84s.\ 3\frac{3}{4}d. + \frac{1}{5}$ of $3\frac{3}{4}d.$, or $\frac{1}{5}$ of $\frac{15}{4}d. = \frac{3}{4}d.$; and $84s.\ 3\frac{3}{4}d. + \frac{3}{4}d. = 84s.\ 4\frac{1}{2}d.$ *Ans.*

[Whenever the given sum consists of pounds only, and has 5 or 0 in the unit's place, we know that its fifth part will be a whole number; and in this case it will save figures, in working by Rule I., to execute the division by 5 first: and the same may be said, whatever be the unit's figure of the pounds, if either 5*s.* or 10*s.* or 15*s.* be connected with those pounds. The work of Ex. 3 above may be thus shortened.]

The foregoing Rules are derived from the truth, that if the principal be expressed in pounds, the commission (or interest), expressed also in pounds, will be found by multi-

plying by the rate per cent., and dividing by 100. But if we take 20 times the number of pounds in the principal, the commission will, of course, be expressed in shillings; and the multiplying by 20 and then dividing by 100 is the same as not multiplying at all, and dividing by 5; so that we have only to regard the principal, in pounds, as so many shillings, and then to divide by 5, to get the commission in shillings; and this is Rule I. Rule II. is an obvious inference from it, for we merely double the multiplier (the rate), and double the divisor (5). And Rule III. is but a slight modification of this: in dividing by 10 we cut off the unit's figure, convert the shillings thus cut off into pence, add these to whatever pence may follow, and then divide by 10; or, expressing the odd pence as a fraction of a shilling, we multiply all that is cut off by 12 and divide by 10; that is, we multiply by $\frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$; which suggests Rule III.

NOTE.—If the commission be 5 per cent., then whatever number of pounds and fractions of a pound there are in the principal, so many shillings and like fractions of a shilling are there in the commission; and the Rule at p. 174 may be employed.

Although in the foregoing Examples we have calculated the commission to the fraction of a farthing—as the Rules lead to results of the most perfect accuracy—yet, in the transactions of actual business, the Commission agent and the Broker usually charge an additional penny for every overplus fractional part of a penny. In the first Example above, the agent's charge would be 33*s.* 3*d.*; in the second, 72*s.* 6*d.*, which it strictly is; and in the third it would be 84*s.* 5*d.*; so that, in working this third Example by Rule III., the supplementary operation for finding what $\frac{1}{5}$ of $3\frac{3}{4}d.$ is, exactly, would, in practice, be omitted. A glance would show that the addition to the $3\frac{3}{4}d.$ cut off would be greater than $\frac{1}{2}d.$ and less than 1*d.*; so that the pence being between 4*d.* and 5*d.*, the charge for commission would be 84*s.* 5*d.*

4. A stockbroker is employed to sell out £536 of Bank Stock: what will be his charge for brokerage, at 2*s.* 6*d.*, that is, £$\frac{1}{8}$ per cent.?

By Rule I.	By Rule II.	By Rule III.
8) 536*s.*	4) 536*s.*	4) 536*s.*
5) 67	1,0) 13,4 = 13*s.* 5*d.* *Ans.*	13,4 = 13*s.* 5*d.* *Ans.*
13*s.* 5*d.* *Ans.*		

The result, by Rule I., is strictly 13*s.* $4\frac{4}{5}d.$; by Rule II. it is 13*s* $4\frac{8}{10}d.$; and by Rule III., 13*s.* 4*d.* + $\frac{4}{5}d.$, the results all agreeing, of course; but the $\frac{4}{5}d.$ or $\frac{8}{10}d.$ is charged as an additional penny.

5. What will be the charge for commission on £127 10*s.*, at $3\frac{1}{2}$ per cent.? *Ans.* £4 9*s.* 3*d.*
6. What is the brokerage upon a money transaction for £385, at 2*s.* 6*d.* per cent.? *Ans.* 9*s.* 8*d.*
7. What sum must be paid for insuring a vessel and cargo, estimated at £2225, at $3\frac{1}{4}$ per cent.? *Ans.* £72 6*s.* 3*d.*

Calculations respecting the Purchase of Freehold Property.

Problem 1.

Given the number of years' purchase (that is the number of years' rent), to find the rate per cent. on the purchase-money.

Rule.—Divide £100 by the number of years' purchase; the quotient will be the rate per cent.

For the rent multiplied by the number of years' purchase is the purchase-money, or principal invested; and the rent itself is the interest received yearly; and as this principal is to £100, so must the interest on the principal (the rent) be to the interest on £100 (the rate per cent.); that is,

Rent × No. of years : £100 : : Rent : to Rate per cent.;

therefore, $\frac{\text{Rent} \times £100}{\text{Rent} \times \text{No. of years}} = \frac{£100}{\text{No. of years}} =$ Rate per cent.;* which is the Rule.

Examples.

1. If 14 years' purchase be given for a freehold estate, what percentage does the purchaser receive per annum?
   £100 ÷ 14 = £7 2*s.* $10\frac{2}{7}$*d.* *Ans.*
2. If freehold property be purchased for 21 years' rental, how much per cent. will the purchaser receive for his investment?
   £100 ÷ 21 = £4 15*s.* $2\frac{2}{7}$*d.* *Ans.*

---

* That the reader may not suppose here that in the expression "Rent × £100," we are implying that money can be multiplied by money, it may be as well to state that what is really implied is that £100 is to be multiplied by the number of pounds in the Rent, or the Rent taken 100 times.

PROBLEM 2.

To determine the rent so that the purchase-money may yield a given rate per cent.

RULE.—Multiply the purchase-money by the given rate per cent., and then divide by 100.

For we have see above that $\frac{\text{Rent} \times 100}{\text{Purchase-money}} = \text{Rate per}$ cent.; therefore, Purchase-money × Rate ÷ 100 = Rent; which is the Rule.

NOTE.—It is obvious, in order that the investment may produce 5 per cent., that the price paid must be 20 years' purchase; and that a 20th part of the purchase-money must be the yearly rental, in order that 5 per cent. may be realised by the holder of the property.

EXAMPLES.

1. If a freehold estate be sold for £30,000, what must be the yearly rent, to allow the purchaser 4 per cent. per annum for his money?
   £300,00 × 4 = £1200. *Ans.*
2. If a freehold estate be sold for £11,000, what must be the yearly rent, to allow the purchaser 5 per cent. per annum?
   £110,00 × 5 = £550. *Ans.*

PROBLEM 3.

The yearly rent and purchase-money being given, to find the rate per cent.

RULE.—100 times the rent divided by the purchase-money will give the rate per cent. [This is obvious from the fraction for "Rate" given above, Prob. 2.]

EXAMPLES.

1. If a freehold yearly rental of £550 be bought for £11,000, at what rate per cent. is the money invested?
   55000 ÷ 11000 = 5 per cent. *Ans.*
2. If an estate of £1200 a year is bought for £30,000, at what rate per cent. is the money invested? *Ans.* 4 per cent.

[This problem differs from Prob. 1 only in this, namely, that here the actual rental is given, and there only the

number of years' purchase: the number of years' purchase being assigned, the yearly rental need not be formally stated, since it is implied.]

We shall only add further on this subject, that since 100 divided by the number of years' purchase gives the rate per cent., it follows that 100 divided by the rate per cent. will give the number of years' purchase; thus, if only 3 per cent. is to be realised, the number of years' purchase for the estate must be $100 \div 3 = 33\frac{1}{3}$ years; if 4 per cent. is to be secured, the number of years' purchase must be $100 \div 4 = 25$ years; and so on. (See Ex. 1, Prob. 2.)

Discount, Stocks.

Discount is analogous to Commission; it is the percentage which the receiver of money allows to the payer for prompt payment; it is also the name given to the deduction which the Banker or Bill-broker makes upon Cashing a Bill or Promissory Note, which Bill or Note becomes due, or payable, only at a future specified time; and the calculation of the Discount on any sum is the same as the calculation of Interest on that sum.

The present worth of such a Bill is not the sum on which the discount is charged; it is that sum of money, paid down, which, when put out at the agreed-upon interest, for the specified time, will amount to just sufficient to pay the Bill when it becomes due: thus, if the Bill be for £105, payable in one year, interest being at 5 per cent. per annum, then, since £100 present money would amount in one year, at the proposed interest, to £105, the present value of the Bill is £100. But Bankers and Bill-discounters reasonably expect a profit, and therefore would charge, as Discount, the full interest of the £105, namely, £5 5*s*. And this would also be the discount, at 5 per cent., allowed to the purchaser of goods by the tradesman, for ready-money payment: in some kinds of purchases, however, $7\frac{1}{2}$, or even 10 per cent., is allowed. After what has been said in reference to interest generally, a single illustrative example here will suffice; it being remembered that three days—called days of grace—are always added to the specified time of payment, so that the Bill is not really due till the third day after that time.

Examples.

1. A Bill for £77, drawn on the 8th of March, at 6 months, is discounted on the 3rd of June, at 5 per cent.: required the amount of discount?

The 6 months expire on September 8; therefore the Bill becomes due September 11. From June 3 to September 3 is 92 days (see Table, p. 26), and therefore to September 11 it is 100 days; and the interest (discount) on £77, at 5 per cent., for 100 days, is found, by the method employed at p. 182, to be £1 1*s.* $1\frac{1}{4}$*d.*; and therefore the discount charged would be £1 1*s.* 2*d.*

The following example belongs to a class of cases of frequent occurrence in Bill transactions.

2. A Bill for £500 was due February 2, 1870, but was allowed to remain at interest. £80 was paid March 9; £115 May 15; £25 June 1; and the balance, namely, £280, August 14: what interest was due at 5 per cent.?

	£		Days.			£
1870; Feb. 2, Due	500	×	35	=	17500	3) 7·1545 (See p. 183.)
Mar. 9, Paid	80					2·3848
						2384
	420	×	67	=	28140	238
May 15 „	115					
						9·8015£
	305	×	17	=	5185	20
June 1 „	25					
						16·03 *s.*
	280	×	74	=	20720	12
Aug. 14 „	280					
					71545	·36 *d.*

The interest charged would be £9 16*s.* 1*d.*, which is the *Ans.*

The Rule at p. 183 directs us to point off five places of figures; whereas, above, we have pointed off only four from 71545: but it is to be remembered that this number is to be previously multiplied by 10,—twice the rate per cent.; so that the number to which the method referred to is to be applied is 715450, from which five figures being pointed off, we have 7·1545, as above, the 0 being omitted as non-significant.

The calculations concerned in the purchase of Stock, that is, property in the Public Funds, are very similar to those employed in the purchase of Freehold Estates. What, in *reference to* this latter kind of property, are called Rents,

in regard to the former kinds are called Dividends; but in each case a permanent yearly or half-yearly income is purchased for a specified sum paid at once. Stock is not, in reality, money; it merely gives the purchaser or holder of it the claim to a certain yearly or half-yearly dividend: it is this right alone that he purchases, and which he can again sell, with but little trouble, whenever he pleases. Like most purchaseable property, or income, the price fluctuates, the more perhaps in this kind of property than in any other kind, since the money invested in the purchase of the income goes to the Government, which, from commercial and political changes, may require, for the exigency of the occasion, more funds at one time than at another, and are therefore willing to sell the incomes, or dividends they grant, at a lower price. The following examples will serve to show the nature of the transactions here spoken of.

Examples.

1. A person invests £3500 in the $3\frac{1}{2}$ Per Cents. when the price of this stock is 98; that is, when he pays £98 for what is called £100 Stock: what will his annual income from this investment be?

He purchases as many £100's Stock as there are 98's in 3500, and for each of these £100's he is to receive £3 10*s.* per annum; therefore his yearly income will be

$$£\frac{3500}{98} \times 3\frac{1}{2} = £\frac{12250}{98} = £125 \textit{ Ans.}$$

2. When the $3\frac{1}{2}$ Per Cents. are at 98, how much money must a person invest in that stock, in order to secure a yearly income of £150?

It is obvious that he must purchase as many £100's Stock as there are £$3\frac{1}{2}$'s in £150; so that we have $150 \div 3\frac{1}{2}$, or $300 \div 7 = 42\frac{6}{7}$; and £$98 \times 42\frac{6}{7}$ = £4200, the sum to be invested. The purchaser will then possess or hold £$4285\frac{5}{7}$ Stock in the $3\frac{1}{2}$ per cents.; since $42\frac{6}{7}$ times £100 amount to this sum, the £98 (the price) being the value of £100 Stock.

3. When the $3\frac{1}{2}$ Per Cents. are at $99\frac{7}{8}$, how much money must be invested in them to produce an income of £140 per annum?
   Ans. £3995.

4. When Bank Stock is at $131\frac{3}{4}$, the interest on it being at 5 per cent., how much money will purchase £575 10*s.* of it; and how much must be paid to the Stockbroker, who charges 2*s.* 6*d.* per cent. on the stock purchased?
   Ans. Purchase-money, £758 4*s.* 5*d.*; Brokerage, 14*s.* 5*d.*

PROFIT AND LOSS.

PROBLEM 1.

The prime cost and the selling price being given, to find the gain or loss per cent.

This problem is solved by a common Rule-of-Three operation. The difference between the cost and the selling price is the gain or loss; and the cost is to £100 as the profit or loss on the cost is to the profit or loss on £100, as is obvious; or without the formality of a Rule-of-Three stating, the Rule may be expressed thus:

RULE.—Divide 100 times the gain or loss by the prime cost; the quotient will be the number of pounds gain or loss per cent.

EXAMPLES.

1. If a horse is bought for £15, and then sold for £17 10*s*., what is the gain per cent.?

The gain on the £15 is £2 10*s*.; and $\frac{250}{15} = \frac{50}{3} = 16\frac{2}{3}$ per cent. *Ans.*

2. If cloth be bought at 6*s*. per yard, and sold at 7*s*., what is the gain per cent.?

The gain on the 6*s*. is 1*s*.; and $\frac{100}{6} = 16\frac{2}{3}$ per cent. *Ans.*

This profit, per cent., is the same as that in the former example: in both cases the gain on the cost is $\frac{1}{6}$ of that cost; so that the calculation in each case might stand thus: $\frac{1}{6} \times 100 = \frac{100}{6} = 16\frac{2}{3}$.

3. If linen be bought at 1*s*. per yard, and sold at $13\frac{1}{2}$*d*., what is the gain per cent.? *Ans.* $12\frac{1}{2}$ per cent.
4. If broadcloth be bought at £1 per yard, and sold at 13*s*. 4*d*., what is the loss per cent.? *Ans.* $33\frac{1}{3}$ per cent.
5. If, on the contrary, the cloth be bought at 13*s*. 4*d*. per yard, and sold at £1, what is the gain per cent.? *Ans.* 50 per cent.
6. If tea be bought at 2*s*. 9*d*. per lb., and sold at 3*s*. 4*d*., what is the gain per cent.? *Ans.* £21 4*s*. $2\frac{10}{11}$*d*. per cent.

PROBLEM 2.

The prime cost being given, to find what the selling price must be, in order that an assigned rate per cent. may be obtained.

RULE.—Whatever part the proposed rate per cent. is of

100, that same part of the cost price will be the requisite profit; this, added to the cost price, is the selling price.

EXAMPLES.

1. A cargo of cotton is bought for £12345; what must it be sold for to yield a profit of 5 per cent.?

	£	
20)	12345	
	617	5*s.*
	£12962	5*s.*

As 5 is the 20th part of 100, we proceed, per Rule, as here annexed; and we thus find the profit to be £617 5*s.*, and the selling-price to be £12962 5*s.* *Ans.*

The reverse of this problem would be to find the prime cost from knowing the profit upon the whole, and the rate of profit per cent. In this case we should have to multiply the given profit by the number which here is the divisor; thus, in the instance of the profit being known to be £617 5*s.*, and 5 the rate per cent., we should have £617 5*s.* $\times$ 20 = £12345 = the prime cost. But we need not extend these examples. Any three of the terms in the general proportion—

Prime cost : £100 : : Gain on outlay : Gain per cent.—

being given, the fourth term may be found, and the figure-work economised, by discarding factors seen to be common to both multiplier and divisor; thus, in the example just considered, the proportion would be—

£100 : Prime cost : : Gain per cent. : Gain on outlay;

that is, £100 : £12345 : : 5 per cent. : £617 5*s.* Gain.

Thus, $\frac{\pounds 12345 \times 5}{100} = \frac{\pounds 12345}{20} = \pounds 617\ 5s.$, as by the Rule.

If the required profit had been at the rate of only 4 per cent., then we should have had—

$$\frac{\pounds 12345 \times 4}{100} = \frac{\pounds 12345}{25} = \pounds 493\ 16s.$$

But it is perhaps more expeditious, in both these, as well as in similar cases, to leave the divisor 100 unreduced, and to proceed thus. Multiplying in the first case by the 5, and in the second by the 4, and cutting off the last two figures

of each product, which is equivalent to dividing by 100, we have—

1st.	£ 617,25	2nd.	£ 493,80
	20		20
	5,00*s.*; £617 5*s.*		16,00*s.*; £493 16*s.*

Proportional Parts.

Problem.

To divide a given quantity into parts which shall have the same relation to one another as any proposed numbers have to one another.

Rule I.—As the sum of the given numbers is to any one of them, so is the given quantity to be divided to the part of it corresponding to that number.

Examples.

1. It is required to divide £80 into three parts, that shall bear to one another the same relations as the numbers 2, 3, and 5.

1st. 10 : 2 :: £80 : £16. 2nd. 10 : 3 :: £80 : £24. 3rd. 10 : 5 :: £80 : £40. The required parts are therefore £16, £24, and £40, which together make up the whole £80. It is obvious that the Rule might be expressed a little differently, thus:—

Rule II.—Multiply the given quantity by each of the given numbers separately, and then divide each product by the sum of all the numbers.

2. A bankrupt owes £120 to A; £80 to B; and £75 to C: he possesses only £165: how is this sum to be equitably divided among his three creditors?

£ 120 ×	£ 165 =	19800,	which ÷	275 =	£ 72,	the share of	A.
80 ×	„ =	13200,	„ ÷	„ =	48,	„	B.
75 ×	„ =	12375,	„ ÷	„ =	45,	„	C.
275					165		

3. Three traders, A, B, and C, contribute the following sums to the business: A. £500; B, £650: and C, £700: the year's profits are £555: what is each partner's share?

Ans. A, £150; B, £195; C, £210.

4. A person bequeathed in his Will £140 to A; 100 guineas to B; 80 guineas to C; £70 to D; and £60 to E: but at his death left only £311 15*s*. How is this sum to be equitably divided, overplus fractions of a farthing being disregarded, because unpayable?
Ans. A, £95 1*s*. $8\frac{3}{4}$*d*.; B, £71 6*s*. $3\frac{1}{2}$*d*.; C, £57 1*s*. $0\frac{1}{2}$*d*.; D, £47 10*s*. $10\frac{1}{4}$*d*.; E, £40 15*s*. $0\frac{1}{4}$*d*.

The truth of the foregoing Rule scarcely requires any formal proof. Take the first Example: here we are told that the number 10 is divided into the three parts, 2, 3, and 5, and we are required to divide the number 80 in a similar way. It is plain that whether the proposed number be twice 10, or three times 10, or any number of times 10 whatever, the required component parts of it must be just as many times 2, 3, and 5; hence the given number (10) must be to the component part of it (2) as the proposed number (80) is to its corresponding component part; and so of each of the other component parts. And similarly in all other such cases.

The Chain-Rule.

The Chain-Rule is a compendious method of computing Examples which, without it, would involve two or more distinct Rule-of-Three statings: the following Examples will sufficiently illustrate the mode of working by it.

Examples.

1. If 3 lbs. of tea cost as much as 8 lbs. of coffee, and 5 lbs. of coffee as much as 18 lbs. of sugar: how many pounds of sugar should be given in exchange for 20 lbs. of tea?

By the Rule-of-Three, we have:—

1st. 8 lbs. coffee : 5 lbs. coffee :: 3 lbs. tea : $\frac{3 \times 5}{8}$ lbs. tea.

2nd. $\frac{3 \times 5}{8}$ lbs. tea : 20 lbs. tea :: 18 lbs. sugar : $\frac{8 \times 18 \times 20}{3 \times 5}$ lbs. of sugar;

since dividing by $\frac{3 \times 5}{8}$ is the same as multiplying by $\frac{8}{3 \times 5}$.

Now, according to the Chain-Rule, the several quantities would be

arranged in two columns, thus:—

$$\begin{aligned} &\text{3 lbs. tea} &&= \text{8 lbs. coffee.} \\ &\text{5 lbs. coffee} &&= \text{18 lbs. sugar.} \\ \text{How many lbs. sugar} &&&= \text{20 lbs. tea?} \end{aligned}$$

Where it is to be observed that no two commodities of the same kind occur in the same column. Now by dividing the product of the numbers in the complete column, by the product of those in the column which the answer, if known, would complete, that answer is obtained: it is $\frac{8 \times 18 \times 20}{3 \times 5} = 192$; so that 192 lbs. of sugar is the answer. And it is plain that the result of the division here directed to be performed must give the true answer, because from the foregoing equalities it follows that we must have also the equality

$$3 \times 5 \times \textit{Ans.} = 8 \times 18 \times 20, \text{ and therefore}$$
$$\textit{Ans.} = \frac{8 \times 18 \times 20}{3 \times 5} = 192.$$

NOTE.—In working Examples by the Chain-Rule, the computer will do well to avail himself of every occasion that may offer to expunge factors common to numerator and denominator—common, that is, to dividend and divisor, before he actually multiplies and divides. For instance, in the present example, he should deal with the fraction $\frac{8 \times 18 \times 20}{3 \times 5}$ in the more simple form $\frac{8 \times 6 \times 4}{1}$, or rather $8 \times 6 \times 4 = 192$. The 8 is retained, intact, in the numerator, because it has no integral factor (or divisor) common to either the 3 or the 5; but the factor (or divisor) 3 is common to the 18 and the 3; this common factor, expunged from both, reduces the 18 to 6 and the 3 to 1. Again; the factor 5, entering into both the 20 and the 5, is in like manner expunged from both numbers,; the 20 being thus reduced to 4, and the 5 to 1. And these simplifications being made mentally, at the outset, the actual figure-work becomes abridged. When a factor is seen to be common to a number in the numerator and a number in the denominator, it will be convenient to draw the pen through both numbers, and to write, above the former and below the latter, only the factor of each which is retained, and then to work with these.

2. If twenty Spanish piastres are worth £3 7*s*. 6*d*.; and £1 be worth $25\frac{1}{3}$ French francs: how many francs may be had in exchange for $2\frac{1}{2}$ Spanish piastres?

$$\begin{aligned} \text{20 piastres} &= \pounds 3\tfrac{3}{8} &&= \frac{\pounds 27}{8} \\ \pounds 1 &= 25\tfrac{1}{3} \text{ francs} &&= \frac{76}{3} \text{ francs} \\ \text{francs?} &= 2\tfrac{1}{2} \text{ piastres} &&= \tfrac{5}{2} \text{ piastres.} \end{aligned}$$

The number of francs is therefore $\frac{27 \times 76 \times 5}{20 \times 8 \times 3 \times 2} = \frac{9 \times 19}{4 \times 2 \times 2}$ (See NOTE above) $= 10\frac{11}{16}$ francs.

3. If £1 = 420*d.* Flemish, and 58*d.* Flemish = 1 Venetian crown = 60 Venetian ducats; 1 ducat = 360 Spanish maravedis, and 272 maravedis = 1 Spanish piastre: how many piastres may be had in exchange for £1000 sterling?

£1	= 420*d.* Flemish	$\frac{420 \times 60 \times 360 \times 1000}{58 \times 272} =$
58*d.* F.	= 60 ducats V.	$575050\frac{350}{493}$ = 575050 piastres,
1 duc. V.	= 360 marav.	and 193 maravedis. *Ans.*
272 marav.	= 1 piastre	
piastres?	= £1000	

[On the general subject of *Exchanges*, consult Kelly's Universal Cambist.]

4. If 3 lbs. of pepper be worth 4 lbs. of mustard, and 5 lbs. of mustard be worth 12 lbs. of candles: how many lbs. of candles should be given for 20 lbs. of pepper? *Ans.* 64 lbs. of candles.

5. If the value of 5 lbs. of tea = 12 lbs. of coffee; 9 lbs. of coffee = 28 lbs. of sugar; and 13 lbs. of sugar = 18 lbs. of soap: how many pounds of soap may be had for 7 lbs. of tea?
Ans. $72\frac{3}{8}$ lbs. of soap very nearly.

The general principle which is the foundation of the Chain-Rule is that of the compounding of equations, according to the following rule:—

Reduce all quantities expressed in more than one term or denomination, either the highest or lowest, with fractions, if any; and bring all quantities of the like kind to one and the same denomination. Let x represent the required unknown quantity, which is sought in resolution of the problem or question; and set down, in a parallel column on the right, as consequent, its given equivalent, as stated in the question. Below the antecedent x, set down the other given term, or quantity of the same kind as the last preceding consequent, accompanied on its right by its own given consequent; and so on, with the remaining given terms, concluding with the consequent which is of the same kind as the first antecedent x. Multiply together all the consequents or right-hand terms of the statement; and similarly all the given antecedents or left-hand terms; perform the division of the former product, as dividend, by the latter product, as divisor, after cancelling all factors common to both. The quotient will be the value of x, and the answer to the question.

EXAMPLE.

What will an estate worth £39420 per annum amount to per minute?

	x	. . . . . .	1 minute
Minutes,	60	. . . . . .	1 hour
Hours,	24	. . . . . .	1 day
Days,	365	. . . . . .	1 year
Year,	1	. . . . . .	£39420
£	1	. . . . . .	20 shillings
Shillings	1	. . . . . .	12 pence

Then $\frac{39420 \times 20 \times 12}{365 \times 24 \times 60} = \frac{2628}{73 \times 2} = 18$ pence or 1*s.* 6*d.* per minute.

THE SLIDE-RULE.

The slide-rule is a kind of logarithmic table, and is so constructed as to obtain the solution of arithmetical questions in either multiplication, division, or extraction of the roots of numbers. It is formed of two pieces of box-wood, each 12 inches in length, joined together by a brass folding joint. In one of the pieces there is a brass slider. The rules are commonly marked with A on the rule, B and C on the slider, and D on the girt or square line. Let the learner observe whatever value is given to the first 1 from the left, the numbers following, viz.—2, 3, 4, 5, &c., will represent twice, thrice, four times, &c., that value. If one is reckoned one or unity, the rest will count 2, 3, 4, &c.; but if the one is reckoned ten, then 2, 3, 4, will count 20, 30, 40, &c. Should the first one be called 100, then 2, 3, 4, &c., will count 200, 300, 400, &c. The value of the 1 in the middle of the line is always ten times that of the first 1; the value of the second 2 is ten times that of the first 2; so that if the value of the first 1 be 10, that of the second 1 will be 100; the first 2 will be 20, and the second 2 will be 200, &c. On the lines A, B, and C, there are 50 small divisions betwixt 1 and 2, 2 and 3, 3 and 4, &c. Now, if the first 1 be reckoned 1 or unity, each of the small divisions between 1 and 2, and 2 and 3, &c., will be $\frac{1}{50}$ or ·02; and if you take the first 1 to be ten, then the small divisions from the second 1 to 2, 2 to 3, &c., will each be ten times

greater than $\frac{1}{50}$, or ·02, each of them will be $\frac{10}{50}$, or $\frac{1}{5}$, or ·2. Again, if 1 represents 100, the first 2 will be 200; if the second 1 be 1000, the second 2 will be 2000, and so on. The above being well understood, we shall now proceed to the use of the rule.

Problem 1.

Multiplication.

Rule 1.—Set 1 on B to one of the factors on A; next against the factor on B, you have the product on A.

Examples.

1. Find the product of 3 by 8.

Direction.—Set 1 on B to 3 on A, then against 8 on B will be found the product 24 on A.

2. Find the product of 34 by 16.

Direction.—Set 1 on B against 16 on A; then look on B for 34, and against it on the line A will be found the product 544.

Problem 2.

Division.

Rule 2.—Set the divisor on B to the dividend on A; against 1 on B you have the quotient on A.

Examples.

1. Find the quotient of 96 divided by 6.

Direction.—Move the slider till 1 on B stands against 6 on A; then the quotient 16 will be found on B, against the dividend 96 on A.

2. What is the quotient of 108 divided by 12?

Direction.—Set 12 on B against 1 on A; on the line A will be found the quotient 9 against 108 on B.

Problem 3.

Proportion.

Rule 3.—Set the first term on the slider B to the second on A; then on the line A will be the fourth term standing against the third term on B.

Example.

1. If 4 lbs. of brass cost 36*d.*, what will 12 lbs. come to?

Direction.—Move the slider so that 4 on B will stand against 12 on A, then against 36 on B will be found the fourth term 108 on A.

Problem 4.

Superficial measure.

Rule 4.—Multiply the length by the breadth, the product will be the area.

Direction.—Set 12 on B against the breadth in inches on A; on the line A will be found the surface in square feet against the length in feet on the line B.

Example.

1. What is the content of a plank 18 in. broad, and 10 ft. 3 in. long?

Direction.—Move the slider so that 12 on B stands against 18 on A; then will $10\frac{1}{4}$ on B stand against $15\frac{3}{8}$ on A, which is $15\frac{3}{8}$ square feet.

Problem 5.

To find the solid content of timber.

Rule 5.—Multiply the length, breadth, and thickness together.

Set the length in feet on C to 12 on D, then on C will be found the content in feet against the square root of the product of the depth and breadth in inches on D.

Example.

1. What is the content of a square log of timber, the length of which is 10 feet, and the side of its square base 15 inches.

Set 10 on C against 12 on D; then will 15 on D stand against the content $15\frac{5}{8}$ on C.

Problem 6.

To extract the square root.

Move the slider so that the middle division on C, which is marked 1, stands against 10 on the line D; then against the given number on C, the square root will be found on D.

Note.—If the given number consists of an even number of places of figures, as 2, 4, 6, &c., it is to be found on the left hand part of the line C; but if odd numbers, as 3, 5, 7, &c., it is to be found on the right hand side of C, 1 being the middle point of the line.

EXAMPLES.

1. Find the square root of 81 ?

The number of places are even, being two; therefore the number 81 is sought for on the left hand side of the line C. Set 1 on C against 10 on D; then against 81 on C will be found 9, the square root on D.

2. What is the square root of 144 ?

Set 1 on C to 10 on D; then against 144 on C will be found the square root 12 on D.

GENERAL REMARKS.

The slide-rule is also a ready resource for the approximate solution, by the quick and easy method of inspection, of a multiplicity of numerical questions, of which the foregoing are only a few examples. Its varied application is almost a study by itself, as an aid to which the student may advantageously consult "The Slide Rule, and How to Use it," by C. Hoare.* As a useful example we supplement this notice by illustrating the method of determining the corresponding equivalents in English and French weights and measures, by reference to standard ratios or gauge-points; A being taken to represent scales or series of French denominations, and B English equivalents:—

On B. Gauge Point.		On A. Gauge Point.
100 inches,	against	254 centimetres.
20 feet,	„	61 decimetres.
35 yards,	„	32 metres.
64 miles,	„	103 kilometres.
31 square inches,	„	200 square centimetres.
183 „ feet,	„	17 „ metres.
61 „ yards,	„	51 „ „
257 acres,	„	104 hectares.
36 cubic inches,	„	590 cubic centimetres.
19 „ feet,	„	538 „ decimetres.
17 „ yards,	„	13 „ metres (steres).
46 imperial gallons,	„	209 litres.
141 „ quarters,	„	410 hectolitres.
571 troy grains,	„	37 grammes.
142 „ lbs.,	„	53 kilogrammes.
97 lbs. avoirdupois	„	44 „
187 tons,	„	190 milliers, (metric tonnes).

In all these cases, opposite any given number (French denomination), on A, will be found the equivalent number (English denomination) on B.

* No. 158 of Weale's Rudimentary Series, published by Crosby Lockwood & Co.

SUPPLEMENTARY TABLES.

TABLE I.—This Table shows the values, expressed in decimals of £1, of all sums from $\frac{1}{4}d.$ up to 20*s*. The following instances will sufficiently exemplify the use of it.

1. Express the value of 11*s*. 5$\frac{3}{4}$*d*. in decimals of £1.
   By the Table (p. 208), we find that 11*s*. 5$\frac{3}{4}$*d*. is £·57395833.

2. Required the value of £7 8*s*. 7$\frac{1}{2}$*d*. in decimals of £1.
   By the Table, 8*s*. 7$\frac{1}{2}$*d*. = £·43125 (p. 208), therefore, £7 8*s*. 7$\frac{1}{2}$*d*. = £7·43125.

3. What is the value, in £ *s*. *d*., of £3·4628? Here we look in the Table for the decimal ·4628, and at page 208 we find the number the nearest to it to be ·4625, against which stands 9*s*. 3*d*.; therefore £3·4628 = £3 9*s*. 3*d*., to the nearest farthing.

To find the exact value of £3·4628 by calculation, we should proceed as in the margin; from which we see that £3·4628 = £3 9*s*. 3*d*. + $\frac{288}{1000}f.$, this latter fraction being a little more than a quarter of a farthing. We know that $\frac{1}{4}f.$ is ·25*f*.; and ·288 is ·038 greater than this; that is, it exceeds $\frac{1}{4}f.$ by $\frac{38}{1000}f.$, which is less than a 25th part of a farthing.

$$\begin{array}{r} £3·4628 \\ 20 \\ \hline 9·256s. \\ 12 \\ \hline 3·072d. \\ 4 \\ \hline ·288f. \\ \hline \end{array}$$

But the value of the small difference between any given decimal of £1, and the approximation to that decimal, in the Table, may be readily calculated, whenever it is thought necessary to do so, by simply converting the small difference into the decimal of a farthing; thus, in the case before us, the difference between the tabular number ·4625 and the given number ·4628 is ·0003; and this multiplied by 20 × 12 × 4 is ·288*f*.

TABLE II.—This table expresses any number of days, from 1 day to 365 days, in decimals of a year, and will be found useful in determining the Interest of any sum of money, at a given rate per cent., for any specified number of days. In using the Table for this purpose, it is necessary to remember that the yearly interest of £1, at any rate per cent., is the 100th part of that rate; that is, it is the rate itself with a cipher prefixed and preceded by the decimal point: thus—The interest of £1, at 2 per cent., at 2$\frac{1}{2}$, at 3, at 3$\frac{1}{2}$, at 4, at 4$\frac{1}{2}$, is £·02, £·025, £·03, £·035, £·04, £·045, and so on. As an instance of the application of the Table, let us take Example 1, at p. 182, the principal being £325 7*s*., the time 89 days, and the interest 4$\frac{1}{2}$ per cent.

Now 7*s.* = £·35; this is, of course, given in Table I.; but there is never any necessity to refer to the Table in the case of shillings only, since we have merely to regard the units figure as a decimal and divide by 2. The given sum is therefore £325·35, and this multiplied by ·045, the rate per cent., gives 14·64075. By Table II., the decimal for 89 days is the number ·24384* (limiting the decimals to five places,—a number quite sufficient), and 14·64075 × ·24384 =3·57. (See NOTE below.) Turning now to Table I., we find that the nearest decimal to ·57 is ·56979, to which corresponds 11*s.* 4¾*d.* Hence the required interest, to the nearest farthing, is £3 11*s.* 4¾*d.*, as at p. 182

NOTE.—Whenever numbers involving several places of decimals are to be multiplied together, or to be divided the one by the other, it is the better way to perform the operations by what are called *Contracted Multiplication* and *Contracted Division;* as taught in most books on common Arithmetic.† The multiplication of 14·64075 by ·24384, indicated above, is here executed by both the common method and the contracted method.

Common method.	*Contracted method.*
14·64075	14·64075
·24384	48342
5856300	2928150
11712600	585630
4392225	43922
5856300	11713
2928150	586
3·5700004800	3·57000

* Whenever any of the terminating decimals are rejected, as being superfluous for the case in hand, the last of the figures retained should always be increased by a unit, if the first of the rejected figures be a 5 or a greater number. It is plain, in the instance in the text, that five decimals from the Table will be a sufficient number to employ; for the error of so much as even a unit in the fifth place would be an error only of $\frac{1}{100000}$; and it is easy to foresee that this part of £14·64075 can be but an insignificant fraction of a farthing; for this part of £100 even is less than a whole farthing.

† See the "ARITHMETIC," in Weale's *Rudimentary Series*, published by CROSBY LOCKWOOD & Co.

TABLE I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
8	0¼	·40104167	9	0¼	·45104167	10	0¼	·50104167	11	0¼	·55104167
8	0½	·40208333	9	0½	·45208333	10	0½	·50208333	11	0½	·55208333
8	0¾	·403125	9	0¾	·453125	10	0¾	·503125	11	0¾	·553125
8	1	·40416667	9	1	·45416667	10	1	·50416667	11	1	·55416667
8	1¼	·40520833	9	1¼	·45520833	10	1¼	·50520833	11	1¼	·55520833
8	1½	·40625	9	1½	·45625	10	1½	·50625	11	1½	·55625
8	1¾	·40729167	9	1¾	·45729167	10	1¾	·50729167	11	1¾	·55729167
8	2	·40833333	9	2	·45833333	10	2	·50833333	11	2	·55833333
8	2¼	·409375	9	2¼	·459375	10	2¼	·509375	11	2¼	·559375
8	2½	·41041667	9	2½	·46041667	10	2½	·51041667	11	2½	·56041667
8	2¾	·41145833	9	2¾	·46145833	10	2¾	·51145833	11	2¾	·56145833
8	3	·4125	9	3	·4625	10	3	·5125	11	3	·5625
8	3¼	·41354167	9	3¼	·46354167	10	3¼	·51354167	11	3¼	·56354167
8	3½	·41458333	9	3½	·46458333	10	3½	·51458333	11	3½	·56458333
8	3¾	·415625	9	3¾	·465625	10	3¾	·515625	11	3¾	·565625
8	4	·41666667	9	4	·46666667	10	4	·51666667	11	4	·56666667
8	4¼	·41770833	9	4¼	·46770833	10	4¼	·51770833	11	4¼	·56770833
8	4½	·41875	9	4½	·46875	10	4½	·51875	11	4½	·56875
8	4¾	·41979167	9	4¾	·46979167	10	4¾	·51979167	11	4¾	·56979167
8	5	·42083333	9	5	·47083333	10	5	·52083333	11	5	·57083333
8	5¼	·421875	9	5¼	·471875	10	5¼	·521875	11	5¼	·571875
8	5½	·42291667	9	5½	·47291667	10	5½	·52291667	11	5½	·57291667
8	5¾	·42395833	9	5¾	·47395833	10	5¾	·52395833	11	5¾	·57395833
8	6	·425	9	6	·575	10	6	·525	11	6	·575
8	6¼	·42604167	9	6¼	·47604167	10	6¼	·52604167	11	6¼	·57604167
8	6½	·42708333	9	6½	·47708333	10	6½	·52708333	11	6½	·57708333
8	6¾	·428125	9	6¾	·478125	10	6¾	·528125	11	6¾	·578125
8	7	·42916667	9	7	·47916667	10	7	·52916667	11	7	·57916667
8	7¼	·43020833	9	7¼	·48020833	10	7¼	·53020833	11	7¼	·58020833
8	7½	·43125	9	7½	·48125	10	7½	·53125	11	7½	·58125
8	7¾	·43229167	9	7¾	·48229167	10	7¾	·53229167	11	7¾	·58229167
8	8	·43333333	9	8	·48333333	10	8	·53333333	11	8	·58333333
8	8¼	·434375	9	8¼	·484375	10	8¼	·534375	11	8¼	·584375
8	8½	·43541667	9	8½	·48541667	10	8½	·53541667	11	8½	·58541667
8	8¾	·43645833	9	8¾	·48645833	10	8¾	·53645833	11	8¾	·58645833
8	9	·4375	9	9	·4875	10	9	·5375	11	9	·5875
8	9¼	·43854167	9	9¼	·48854167	10	9¼	·53854167	11	9¼	·58854167
8	9½	·43958333	9	9½	·48958333	10	9½	·53958333	11	9½	·58958333
8	9¾	·440625	9	9¾	·490625	10	9¾	·540625	11	9¾	·590625
8	10	·44166667	9	10	·49166667	10	10	·54166667	11	10	·59166667
8	10¼	·44270833	9	10¼	·49270833	10	10¼	·54270833	11	10¼	·59270833
8	10½	·44375	9	10½	·49375	10	10½	·54375	11	10½	·59375
8	10¾	·44479167	9	10¾	·49479167	10	10¾	·54479167	11	10¾	·59479167
8	11	·44583333	9	11	·49583333	10	11	·54583333	11	11	·59583333
8	11¼	·446875	9	11¼	·496875	10	11¼	·546875	11	11¼	·596875
8	11½	·44791667	9	11½	·49791667	10	11½	·54791667	11	11½	·59791667
8	11¾	·44895833	9	11¾	·49895833	10	11¾	·54895833	11	11¾	·59895833
9	0	·45	10	0	·5	11	0	·55	12	0	·6

Table I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
12	0¼	·60104167	13	0¼	·65104167	14	0¼	·70104167	15	0¼	·75104167
12	0½	·60208333	13	0½	·65208333	14	0½	·70208333	15	0½	·75208333
12	0¾	·603125	13	0¾	·653125	14	0¾	·703125	15	0¾	·753125
12	1	·60416667	13	1	·65416667	14	1	·70416667	15	1	·75416667
12	1¼	·60520833	13	1¼	·65520833	14	1¼	·70520833	15	1¼	·75520833
12	1½	·60625	13	1½	·65625	14	1½	·70625	15	1½	·75625
12	1¾	·60729167	13	1¾	·65729167	14	1¾	·70729167	15	1¾	·75729167
12	2	·60833333	13	2	·65833333	14	2	·70833333	15	2	·75833333
12	2¼	·609375	13	2¼	·659375	14	2¼	·709375	15	2¼	·759375
12	2½	·61041667	13	2½	·66041667	14	2½	·71041667	15	2½	·76041667
12	2¾	·61145833	13	2¾	·66145833	14	2¾	·71145833	15	2¾	·76145833
12	3	·6125	13	3	·6625	14	3	·7125	15	3	·7625
12	3¼	·61354167	13	3¼	·66354167	14	3¼	·71354167	15	3¼	·76354167
12	3½	·61458333	13	3½	·66458333	14	3½	·71458333	15	3½	·76458333
12	3¾	·615625	13	3¾	·665625	14	3¾	·715625	15	3¾	·765625
12	4	·61666667	13	4	·66666667	14	4	·71666667	15	4	·76666667
12	4¼	·61770833	13	4¼	·66770833	14	4¼	·71770833	15	4¼	76770833
12	4½	·61875	13	4½	·66875	14	4½	·71875	15	4½	·76875
12	4¾	·61979167	13	4¾	·66979167	14	4¾	·71979167	15	4¾	·76979167
12	5	·62083333	13	5	·67083333	14	5	·72083333	15	5	·77083333
12	5¼	·621875	13	5¼	·671875	14	5¼	·721875	15	5¼	·771875
12	5½	·62291667	13	5½	·67291667	14	5½	·72291667	15	5½	·77291667
12	5¾	·62395833	13	5¾	·67395833	14	5¾	·72395833	15	5¾	·77395833
12	6	·625	13	6	·675	14	6	·725	15	6	·775
12	6¼	·62604167	13	6¼	·67604167	14	6¼	·72604167	15	6¼	·77604167
12	6½	·62708333	13	6½	·67708333	14	6½	·72708333	15	6½	·77708333
12	6¾	·628125	13	6¾	·678125	14	6¾	·728125	15	6¾	·778125
12	7	·62916667	13	7	·67916667	14	7	·72916667	15	7	·77916667
12	7¼	·63020833	13	7¼	·68020833	14	7¼	·73020833	15	7¼	·78020833
12	7½	·63125	13	7½	·68125	14	7½	·73125	15	7½	·78125
12	7¾	·63229167	13	7¾	·68229167	14	7¾	·73229167	15	7¾	·78229167
12	8	·63333333	13	8	·68333333	14	8	·73333333	15	8	·78333333
12	8¼	·634375	13	8¼	·684375	14	8¼	·734375	15	8¼	·784375
12	8½	·63541667	13	8½	·68541667	14	8½	·73541667	15	8½	·78541667
12	8¾	·63645833	13	8¾	·68645833	14	8¾	·73645833	15	8¾	·78645833
12	9	·6375	13	9	·6875	14	9	·7375	15	9	·7875
12	9¼	·63854167	13	9¼	·68854167	14	9¼	·73854167	15	9¼	·78854167
12	9½	·63958333	13	9½	·68958333	14	9½	·73958333	15	9½	·78958333
12	9¾	·640625	13	9¾	·690625	14	9¾	·740625	15	9¾	·790625
12	10	·64166667	13	10	·69166667	14	10	·74166667	15	10	·79166667
12	10¼	·64270833	13	10¼	·69270833	14	10¼	·74270833	15	10¼	·79270833
12	10½	·64375	13	10½	·69375	14	10½	·74375	15	10½	·79375
12	10¾	·64479167	13	10¾	·69479167	14	10¾	·74479167	15	10¾	·79479167
12	11	·64583333	13	11	·69583333	14	11	·74583333	15	11	·79583333
12	11¼	·646875	13	11¼	·696875	14	11¼	·746875	15	11¼	·796875
12	11½	·64791667	13	11½	·69791667	14	11½	·74791667	15	11½	·79791667
12	11¾	·64895833	13	11¾	·69895833	14	11¾	·74895833	15	11¾	·79895833
13	0	·65	14	0	·7	15	0	·75	16	0	·8

TABLE II.—*continued.* Decimals of a Year.

Days.	Years.	Days.	Years.	Days.	Years.	Days.	Years.
201	·55068493	251	·68767123	301	·82465753	351	·96164384
202	·55342466	252	·69041096	302	·82739726	352	·96438356
203	·55616438	253	·69315068	303	·83013699	353	·96712329
204	·55890411	254	·69589041	304	·83287671	354	·96986301
205	·56164384	255	·69863014	305	·83561644	355	·97260274
206	·56438356	256	·70136986	306	·83835616	356	·97534247
207	·56712329	257	·70410959	307	·84109589	357	·97808219
208	·56986301	258	·70684932	308	·84383562	358	·98082192
209	·57260274	259	·70958904	309	·84657534	359	·98356164
210	·57534247	260	·71232877	310	·84931507	360	·98630137
211	·57808219	261	·71506849	311	·85205479	361	·98904110
212	·58082192	262	·71780822	312	·85479452	362	·99178082
213	·58356164	263	·72054795	313	·85753425	363	·99452055
214	·58630137	264	·72328767	314	·86027397	364	·99726027
215	·58904110	265	·72602740	315	·86301370	365	1·
216	·59178082	266	·72876712	316	·86575342		
217	·59452055	267	·73150685	317	·86849315		
218	·59726027	268	·73424658	318	·87123288		
219	·60000000	269	·73698630	319	·87397260		
220	·60273973	270	·73972603	320	·87671233		
221	·60547945	271	·74246575	321	·87945205		
222	·60821918	272	·74520548	322	·88219178		
223	·61095890	273	·74794521	323	·88493151		
224	·61369863	274	·75068493	324	·88767123		
225	·61643836	275	·75342466	325	·89041096		
226	·61917808	276	·75616438	326	·89315068		
227	·62191781	277	·75890411	327	·89589041		
228	·62465753	278	·76164384	328	·89863014		
229	·62739726	279	·76438356	329	·90136986		
230	·63013699	280	·76712329	330	·90410959		
231	·63287671	281	·76986301	331	·90684932		
232	·63561644	282	·77260274	332	·90958904		
233	·63835616	283	·77534247	333	·91232877		
234	·64109589	284	·77808219	334	·91506849		
235	·64383562	285	·78082182	335	·91780822		
236	·64657534	286	·78356164	336	·92054795		
237	·64931507	287	·78630137	337	·92328767		
238	·65205479	288	·78904110	338	·92602740		
239	·65479452	289	·79178082	339	·92876712		
240	·65753425	290	·79452055	340	·93150685		
241	·66027397	291	·79726027	341	·93424658		
242	·66301370	292	·80000000	342	·93698630		
243	·66575342	293	·80273973	343	·93972603		
244	·66849315	294	·80547945	344	·94246575		
245	·67123288	295	·80821918	345	·94520548		
246	·67397260	296	·81095890	346	·94794521		
247	·67671233	297	·81369863	347	·95068493		
248	·67945205	298	·81643836	348	·95342466		
249	·68219178	299	·81917808	349	·95616438		
250	·68493151	300	·82191781	350	·95890411		

TABLE III.—TROY WEIGHT.

A Lb. the Integer.	Decimal Parts of 1 lb. Troy.
1 grain . .	lb. 0·0001736
2 grains . .	·0003472
3 „ . .	·0005208
4 g., or 1 carat.	·0006944
2 carats . .	·0013889
3 „ . .	·0020833
4 „ . .	·0027778
5 „ . .	·0034722
6 c., or 1 dwt.	·0041667
2 dwts. . .	·0083333
3 „ . .	·0125000
4 „ . .	·0166667
5 „ . .	·0208333
6 „ . .	·0250000
7 „ . .	·0291667
8 „ . .	·0333333
9 „ . .	·0375000
10 „ . .	·0416667
11 „ . .	·0458333
12 „ . .	·0500000
13 „ . .	·0541667
14 „ . .	·0583333
15 „ . .	·0625000
16 „ . .	·0666667
17 „ . .	·0708333
18 „ . .	·0750000
19 „ . .	·0791667
20 dwts., or 1 oz	·0833333
2 oz. . .	·1666667
3 „ . .	·2500000
4 „ . .	·3333333
5 „ . .	·4166667
6 „ . .	·5000000
7 „ . .	·5833333
8 „ . .	·6666667
9 „ . .	·7500000
10 „ . .	·8333333
11 „ . .	·9166667
12 „ . .	1·0000000

TABLE IV.—AVOIRDUPOIS.

A lb. the Integer.	Decimal Parts.
1 drachm .	lb. 0·0039062
2 drachms .	·0078125
3 „ . .	·0117187
4 „ . .	·0156250
5 „ . .	·0195312
6 „ . .	·0234375
7 „ . .	·0273437
8 „ . .	·0312500
9 „ . .	·0351562
10 „ . .	·0390625
11 „ . .	·0429687
12 „ . .	·0468750
13 „ . .	·0507812
14 „ . .	·0546875
15 „ . .	·0585937
16 d., or 1 oz. .	·0625000
2 oz. . .	·1250000
3 „ . .	·1875000
4 „ . .	·2500000
5 „ . .	·3125000
6 „ . .	·3750000
7 „ . .	·4375000
8 „ . .	·5000000
9 „ . .	·5625000
10 „ . .	·6250000
11 „ . .	·6875000
12 „ . .	·7500000
13 „ . .	·8125000
14 „ . .	·8750000
15 „ . .	·9375000
16 oz., or 1 lb. .	1·0000000

NOTE.—Table IV. in Avoirdupois Weight is used in small quantities, and Table V. is constructed for large ones.

Table V.—Avoirdupois.

A Cwt. the Integer.	Decimal Parts of 1 Cwt.
	cwt.
1 drachm	0·0000348
2 drachms	·0000697
3 ,,	·0001046
4 ,,	·0001395
5 ,,	·0001743
6 ,,	·0002092
7 ,,	·0002441
8 ,,	·0002790
9 ,,	·0003139
10 ,,	·0003487
11 ,,	·0003836
12 ,,	·0004185
13 ,,	·0004534
14 ,,	·0004882
15 ,,	·0005231
16 d., or 1 oz.	·0005580
2 oz.	·0011160
3 ,,	·0016741
4 ,,	·0022321
5 ,,	·0027901
6 ,,	·0033482
7 ,,	·0039062
8 ,,	·0044642
9 ,,	·0050223
10 ,,	·0055803
11 ,,	·0061383
12 ,,	·0066964
13 ,,	·0072544
14 ,,	·0078125
15 ,,	·0083705
16 oz., or 1 lb.	·0089285
2 lbs.	·0178571
3 ,,	·0267857
4 ,,	·0357142
5 ,,	·0446428
6 ,,	·0535714
7 ,,	·0625000
8 ,,	·0714286
9 ,,	·0803571
10 ,,	·0892857
11 ,,	·0982142
12 ,,	·1071428
13 ,,	·1160714
14 lbs., or 1 st.	·1250000
1 qr.	·2500000
2 qrs.	·5000000
3 ,,	·7500000
4 ,,	1·0000000

Table VI.—Chemists' Weight.

A Lb. the Integer.	Decimal Parts.
1 grain	lb. 0·0001736
2 grains	·0003472
3 ,,	·0005208
4 ,,	·0006944
5 ,,	·0008680
6 ,,	·0010417
7 ,,	·0012153
8 ,,	·0013888
9 ,,	·0015625
10 ,,	·0017361
11 ,,	·0019097
12 ,,	·0020833
13 ,,	·0022569
14 ,,	·0024306
15 ,,	·0026042
16 ,,	·0027778
17 ,,	·0029514
18 ,,	·0031250
19 ,,	·0032986
20 gr. or 1 scr.	·0034722
2 scruples	·0069444
3 ditto, or 1 dr.	·0104167
2 drachms	·0208333
3 ,,	·0312500
4 ,,	·0416667
5 ,,	·0520833
6 ,,	·0625000
7 ,,	·0729167
8 drs., or 1 oz.	·0833333
2 oz.	·1666667
3 ,,	·2500000
4 ,,	·3333333
5 ,,	·4166667
6 ,,	·5000000
7 ,,	·5833333
8 ,,	·6666667
9 ,,	·7500000
10 ,,	·8333333
11 ,,	·9166667
12 ,, or 1 lb.	1·0000000

Table VII.—Long Measure.

A Mile the Integer.	Decimal Parts.
1 barleycorn .	M. 0·0000052
2 barleycorns .	·0000105
3 ditto, or 1 inch . .	·0000157
2 inches . .	·0000315
3 ,, . .	·0000473
4 ,, or 1 hand . .	·0000631
5 inches . .	·0000789
6 ,, . .	·0000947
7 ,, . .	·0001104
8 ,, . .	·0001262
9 ,, . .	·0001420
10 ,, . .	·0001578
11 ,, . .	·0001736
12 ,, or 1 foot . .	·0001894
2 feet . .	·0003788
3 ,, or 1 yard	·0005682
2 yards . .	·0011364
3 ,, . .	·0017046
4 ,, . .	·0022727
5½ ,, .	·0028409
1 pole .	·0031250
2 poles . .	·0062500
3 ,, . .	·0093750
4 poles, or 1 landchain .	·0125000
2 chains . .	·0250000
3 ,, . .	·0375000
4 ,, . .	·0500000
5 ,, . .	·0625000
6 ,, . .	·0750000
7 ,, . .	·0875000
8 ,, . .	·1000000
9 ,, . .	·1125000
10 ,, or 1 furlong .	·1250000
2 furlongs .	·2500000
3 ,, .	·3750000
4 ,, .	·5000000
5 ,, .	·6250000
6 ,, .	·7500000
7 ,, .	·8750000
8 ,, or 1 mile . .	1·0000000

Table VIII.—Cloth Measure.

A Yard the Integer.	Decimal Parts.
¼ inch . . .	yd. 0·0069444
½ ,, . . .	·0138889
¾ ,, . . .	·0208333
1 ,, . . .	·0277777
2 ,, . . .	·0555555
1 nail . . .	·0625000
2 nails . . .	·1250000
3 ,, . . .	·1875000
4 ,, or 1 quarter	·2500000
2 quarters . .	·5000000
3 ,, . .	·7500000
4 ,, or 1 yard	1·0000000

Table IX.—Ale, Beer, Spirit, and Wine Measure.

A Gallon the Integer.	Decimal Parts.
1 gill . . .	gal. 0·0312500
2 gills . . .	·0625000
3 ,, . . .	·0937500
4 ,, or 1 pint .	·1250000
2 pints, or 1 quart .	·2500000
2 quarts . . .	·5000000
3 ,, . . .	·7500000
4 ,, or 1 gallon	1·0000000

Table X.—Dry Measure.

A Quarter the Integer.	Decimal Parts.
1 pint . . .	qr. 0·0019531
2 pints, or 1 quart .	·0039062
2 qts., or 1 pottle .	·0078125
2 pottles, or 1 gallon	·0156250
2 gallons, or 1 peck	·0312500
2 pecks . . .	·0625000
3 ,, . . .	·0937500
4 ,, or 1 bushel	·1250000
2 bushels . .	·2500000
3 ,, . .	·3750000
4 ,, . .	·5000000
5 ,, . .	·6250000
6 ,, . .	·7500000
7 ,, . .	·8750000
8 ,, or 1 quarter	1·0000000

FRENCH WEIGHTS, MEASURES, AND MONEY, WITH THEIR ENGLISH EQUIVALENTS.

1. WEIGHTS.

[The French unit of weight is the GRAMME = 15·432325 grains English. It is the weight of a cubic centimètre of distilled water.]

Milligramme	= 1000th of a gramme	. . . = ·01543 grains English.	
Centigramme	= 100th „	. . . = ·1543 „	
Décigramme	= 10th „	. . = 1·5432 „	
GRAMME	. . . .	. = 15·4323 „	
Décagramme	= 10 grammes	. = 154·3232 „	
Hectogramme	= 100 „	. = 1543·2325 „	
Kilogramme	= 1000 „	. = $32\frac{1}{8}$ oz. Troy	= 2·2046 lbs. av.
Myriagramme	= 10000 „	= $321\frac{3}{8}$ oz. „	= 22·046 „

*** 51 Kilogrammes make 1 cwt. and very nearly $\frac{1}{2}$ lb. besides.

To convert Avoirdupois into metrical weight.

1 grain	=	0·0648 grammes.
1 dram	=	1·7718 „
1 ounce	=	28·3495 „
1 pound	=	0·4536 kilogrammes.
1 stone	=	6·3503 „
1 quarter	=	12·7006 „
1 hundredweight	=	50·8024 „
1 ton	=	1016·0475 „

To convert Troy into metrical weight.

1 grain	=	0·0648 grammes.
1 pennyweight	=	1·5552 „
1 ounce	=	31·1035 „
1 pound	=	0·3732 kilogrammes.

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2. MEASURES.

Length.

[The French unit of linear measure is the MÈTRE = *39·3708* inches. It is the 10 millionth part of the arc of the *meridian* from the equator to the pole.]

Millimètre	= 1000th of a mètre	. . . =	·03937 inches.
Centimètre	= 100th ,,	. . . =	·39371 ,,
Décimètre	= 10th ,,	. . =	3·93708 ,,
Mètre	. . . .	. . =	39·3708 = 3·2809 ft.
Décamètre	= 10 mètres	. . =	32·809 ft. = 10·9363 yds.
Hectomètre	= 100 ,,	. =	328·09 ft. = 109·363 yds.
Kilomètre	= 1000 ,,	. =	1093·63 yds. = ·62138 miles.
Myriamètre	= 10000 ,,	. =	10936·33 yds. = 6·21382 miles.

Note 1.—Since the fraction $\frac{5}{8}$ is equal to the decimal ·625, the French kilomètre differs but little from the $\frac{5}{8}$ths of an English mile; the difference being ·625 — ·62138 = ·00362, which is less than the $\frac{4}{1000}$th, or the $\frac{1}{250}$th of a mile; so that by estimating a kilomètre at $\frac{5}{8}$ths of an English mile, we make an error, in excess, of less than 1 mile in 250 miles. For the ordinary purposes of comparison therefore we may regard 8 kilomètres as equal to five miles; so that the distance between any two places, expressed in kilomètres, may be converted into English miles, near enough for general itinerary objects, by multiplying the number of kilomètres by 5, and then dividing the product by 8; as in the instance in the margin, where we see that 40 kilomètres make 25 miles.

$$\begin{array}{r} 40 \text{ kilos.} \\ 5 \\ \hline 8)\ 200 \\ \hline 25 \text{ miles.} \end{array}$$

2. Certain French linear measures have been abolished since the year 1840. The principal of these are the following, frequently mentioned in books anterior to that date.

The French inch (*pouce*) = 12 lines (*lignes*) = 1·094 inches.
,, foot (*pied*) = 12 pouces . . = 13·124 ,,
,, toise = 2 mètres = 6 pieds . . = 6·5618 feet.

The old French itinerary league (*lieue*) was 2·4222 English miles; that is, $2\frac{3}{7}$ miles very nearly.

To convert English into metrical measures of length.

1 inch	=	0·0254 mètres.
1 link	=	0·2012 ,,
1 foot	=	0·3048 ,,
1 yard	=	0·9144 ,,
1 fathom	=	1·8288 ,,
1 pole or rod	=	5·0291 ,,
1 chain	=	20·1164 ,,
1 furlong	=	201·1644 ,,
1 mile	=	1·6093 kilomètres.
1 nautic mile	=	1·9119 ,,

Surface.

The French unit of superficial measure is the Are =

119·603 sq. yds. It is the square of 10 mètres; that is, of a décamètre.]

Centiare	**= 100th of an *are* = 1 square mètre**	**=**	**1·196 sq. yds.**
Are	**. . . . .**	**=**	**119·603 „**
Decare	**= 10 ares . . . . .**	**=**	**·2471 acres.**
Hectare	**= 100 ares . . . . . .**	**=**	**2·4711 „**

To convert English into metrical square measures.

1 sq. inch	**=**	**6·4514 sq. centimètres.**
1 „ foot	**=**	**9·2900 sq. decimètres.**
1 „ yard	**=**	**0·8361 sq. mètres.**
1 „ pole	**=**	**25·2919 „**
1 rood	**=**	**10·1168 ares.**
1 acre	**=**	**0·4047 hectares.**
1 square mile	**=**	**258·9894 „**

Capacity.

[The French unit of capacity is the Litre = 61·02705 cubic inches. It is the cube of one-tenth of a mètre, that is, of a décimètre.]

Millitre	= 1000th of a litre = 1 cubic centimètre	=	·06103 cub. in.
Centilitre	= 100th „	=	·6103 „
Décilitre	= 10th „	=	6·1027 „
Litre	. . . .	=	61·02705 „
Décalitre	= 10 litres	=	610·2705 „ = 2·201 gallons.
Hectolitre	= 100 litres . . .	=	3·53166 cub. ft. = 22·0097 gal.
Kilolitre	= 1000 „ = a c. mètre	=	35·31658 „ = 220·0967 „
Myrialitre	= 10000 „ . .	=	353·1658 „ = 2200·9668 „

In the measurement of solids, a cubic mètre is called a stère, a 10th part of which is a decistère, and 10 stères are a decastère.

To convert English into metrical measures of capacity.

1 gill	=	0·1420 litres.
1 pint	=	0·5679 „
1 quart	=	1·1359 „
1 gallon	=	4·5435 „
1 peck	=	9·0869 „
1 bushel	=	36·3477 „
1 quarter	=	2·9078 hectolitres.

To convert English into metrical measures of solidity.

1 cubic inch	=	16·3862 c. centimètres.
1 cubic foot	=	28·3153 c. décimètres.
1 cubic yard	=	0·7645 c. mètres.

It will be seen, from the preceding Tables of Weights and Measures, that the Mètre, the unit of length, is an element entering into even the system of weights, as well as into linear, superficial, solid, and quantitative measurements. It is on this account that the French system of weights and measures is called the METRICAL SYSTEM: it is at the same time a *Decimal* system; because, proceeding from the fundamental unit, the ascending gradations are uniformly at a tenfold rate, and the descending gradations are uniformly by *tenths*.

The money denominations too, as will appear from the Table next following, are likewise according to the decimal system. Accounts, however, are usually kept, not in francs, décimes, and centimes, but in francs and centimes only.

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3. MONEY.

[The French unit of money is the FRANC = 9·4 pence sterling.]

Centime = 100th of a franc	=	0*s*. ·094*d*.
Décime = 10th ,,	=	0*s*. ·94*d*.
FRANC	=	0*s*. 9·4*d*.
5 franc piece (silver, *and* gold)	=	3*s*. 11*d*.
10 franc piece (gold)	=	7*s*. 11*d*.
20 franc piece or Napoleon (gold)	=	15*s*. 10*d*.

[Among the money denominations now disused, were the *Sol* or *Sou*, half a décime, or the 20th part of a franc; the *Ecu* = 6 francs; and the *Louis d' Or* = 24 francs. The franc was formerly called a *livre*. It is proper to add, however, that the half-décime continues to be a current coin, and bears the inscription CINQ CENTIMES (five centimes), and that it is still commonly called a *sou*.]

NOTE.—In the above Table, the value of each of the several French coins, in English money, is the intrinsic value; that is, the value as respects the weight and fineness of the metal; but the exchangeable values of the coins of the one country for those of the other are regulated by additional considerations,—political and commercial. The £1 or 20*s*. sterling, exchanges, in general, for only 25 francs, which is less than would be given according to the foregoing Table. This number, 25, facilitates the conversion of francs into their exchangeable equivalent in pounds, and the converse; inasmuch as that to divide a number by 25 is the same as to multiply the number by 4 and then divide

the product by 100, this latter division being effected by merely pointing off the last two figures for decimals. We thus have the following Rules.

RULE I.—*To convert francs into their equivalent in pounds sterling.* Point off the last two integral figures for decimals, and then multiply the number by 4: the product will express the equivalent in pounds.

RULE II.—*To convert pounds sterling into their equivalent in francs.* Add two ciphers to the integer number denoting the pounds, and then divide by 4: the quotient will express the equivalent number of francs.

NOTE.—It is to be understood that the number, denoting the francs, to be converted into pounds, does not itself involve decimals of a franc: if it do, the pointing off of the last two integral figures will be the advancing of the decimal point two places to the left.

The number, too, denoting the pounds, to be exchanged for francs, is, in like manner, considered to be an integral number; but if it involve decimals of a pound, the decimal point is to be removed two places to the right; the wanting place, if there be but one place of decimals in the pounds, is to be supplied by a cipher. We shall give an example of two of these conversions.

1. How many pounds are there in 2500 francs?
   $25 \cdot 00 \times 4 = £100$, the *Ans.*
2. How many pounds are there in 3684 francs?
   $36 \cdot 84 \times 4 = £147 \cdot 36 = £147\ 7s.\ 2\frac{2}{5}d.$, the *Ans.*
3. How many francs are there in £120?
   $12000 \div 4 = 3000$ fr., the *Ans.*
4. How many francs are there in £147·36?
   $14736 \div 4 = 3684$ fr., the *Ans.*
5. How many francs are there in £238·648? $23864 \cdot 8 \div 4 = 5966 \cdot 2$ fr.
   = 5966 francs, 2 décimes, or 5966 francs, 20 centimes. *Ans.*

The foregoing are the common Rules for reducing francs to pounds, and pounds to francs: we shall now give a different mode of proceeding; one which will be found to be more especially convenient when the sums proposed are of but small amount.

Since 25 francs = 20*s.*, it follows that any number of francs are equivalent to that same number of shillings diminished by one-fifth of the number; and that any number of shillings are equivalent to that number of francs *and one-fourth* of the number more. Thus:—

1. 125 fr. $=$ 125 $-$ 25 shillings $=$ 100*s*. 2. 37 fr. $=$ 37*s*. $-$ $7\frac{2}{5}$*s*. $=$ $29\frac{3}{5}$*s*. $=$ £1 9*s*. $7\frac{1}{5}$*d*. 3. (Ex. 2 above.) 3684 fr. $=$ 3684*s*. $-$ $736\frac{4}{5}$*s*. $=$ $2947\frac{1}{5}$*s*. $=$ £147 7*s*. $2\frac{2}{5}$*d*. 4. 20 fr. $=$ 16*s*. 5. 25 fr. $=$ 20*s*. 6. 1 fr. $=$ 1*s*. $-$ $\frac{1}{5}$*s*. $=$ $9\frac{3}{5}$*d*.

Again 1. 14*s*. $=$ 14 fr. $+$ $3\frac{1}{2}$ fr. $=$ 17 fr. 50c. 2. 17*s*. $=$ 17 fr. $+$ $4\frac{1}{4}$ fr. $=$ 21 fr. 25c. 3. £5 $=$ 100*s*. $=$ 125 fr. 4. £19 $=$ 380*s*. $=$ 380 fr. $+$ 95 fr. $=$ 475 fr. 5. 13*s*. 6*d*. $=$ $13\frac{1}{2}$ fr. $+$ $3\frac{3}{8}$ fr. $=$ $16\frac{7}{8}$ fr. $=$ 16 fr. $87\frac{1}{2}$c. 6. 1*s*. $=$ $1\frac{1}{4}$ fr. $=$ 1 fr. 25c. And even from this the equivalent of any number of shillings may be readily found: thus, 14*s*. $=$ 14 fr. $+$ 250c. $+$ 100c. $=$ 17 fr. 50c.; by regarding 14 times 25c. as 10 times and 4 times.

To what has now been said respecting the comparative values of the French and English coins, we may add, for the practical purposes of visitors to France, that in all the ordinary money transactions of every-day life, ten centimes count as one penny, and five as one halfpenny: thus, 75c. $=$ $7\frac{1}{2}$*d*.; 25c. $=$ $2\frac{1}{2}$*d*.; 15c. $=$ $1\frac{1}{2}$*d*.; and so on.

THE END.

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